Pronouns and Variable Assignments

Lecture 20

April 2, 2024

Announcements:

This lecture is supplemented by the following readings:

• Heim & Kratzer: Ch.4 (110–112)

Your sixth homework assignment is available and is due on Thursday, April 4th.

1 Introduction

We currently have a semantic system for interpreting sentences that relies on the following set of rules:

(1) Functional Application (FA)

If X is a node that has two daughters, Y and Z, and if [Y] is a function whose domain contains [Z], then [X] = [Y] ([Z]).

(2) **Predicate Modification (PM)**

If X is a node that has two daughters, Y and Z, and if $[\![Y]\!]$ and $[\![Z]\!]$ are in $D_{\langle e,t\rangle}$, then $[\![X]\!] = [\lambda x : x \in D_e$. $[\![Y]\!](x) = T$ and $[\![Z]\!](x) = T$

(3) Non-Branching Nodes (NN) Rule

If X is a non-branching node that has Y as its daughter, then [X] = [Y]

(4) Terminal Nodes (TN) Rule

If X is a terminal node, then [X] is specified in the lexicon.

We have focused lately on the ways in which the **context** of an expression can affect its meaning.

During our investigation of determiners and quantifiers, we saw that the denotation of a definite description can depend upon which entities are contextually salient.

- (5) $[\![\text{the }]\!] = [\lambda f_{\langle e,t \rangle} : \text{there is a unique } x \in C \text{ s.t. } f(x) = T \text{ . the unique } x \text{ s.t. } f(x) = T]$
- (6) [the professor] = Sam (in the context of LIN 4470) = Jason (in the context of LIN 4307)

Consequently, the truth conditions of a sentence containing a definite description can vary depending on the context:

(7) [The professor sings] =
$$T$$
 iff (in LIN 4470) Sam sings
= T iff (in LIN 4307) Jason sings

During our recent investigation of implicatures, we saw that it is possible to understand implicatures to represent deductive inferences that:

- (i) arise from the fact that an expression S was asserted in a particular context, and
- (ii) are validated by the assumption that the speaker is observing the Maxims of Conversation.

Thus, as particular or general aspects of the context change, so can the implicatures that are associated with a given utterance.

- (8) A: Are you going to the party tonight?
 - B: I have homework.
 - → I'm not going to the party tonight.
- (9) A: What are you doing?
 - B: I have homework.
 - $\not\rightarrow$ I'm not going to the party.

During our next couple of lectures we will look at another domain in which the meaning expressed by an utterance can depend on the context.

In particular, we will investigate another case in which the truth conditions of an utterance are dependent on the context: **pronouns**.

(10) Context: The speaker is pointing at Sam. (11) Context: The speaker is pointing at Jason.
$$[He \text{ sings }] = T$$
 iff Sam sings $[He \text{ sings }] = T$ iff Jason sings

The overarching goal for the upcoming lectures will be to expand our semantic system to compute the meaning of utterances that contain pronouns. This will require that we capture the fact that the truth conditions of a sentence containing a pronoun is dependent upon the context in which it appears.

These goal will lead us to introduce a new piece of theoretical machinery referred to as a **variable assignment function**. Put informally, this is a contextually-supplied function that maps **indices** to their referents in the discourse

(12) $[X]^g$ = the extension of X relative to the variable assignment function g.

This will allow us to introduce the following **Pronouns Rule** to compute the meaning of pronouns on a context-by-context basis.

(13) **Pronouns Rule (PR)**

If X is a pronoun bearing an index n, then, for any variable assignment g, if n is in the domain of g, then $[X]^g = g(n)$.

As we will see, this is will also require some minor adjustments to our other rules for semantic interpretation.

2 The Problem of Pronouns

2.1 A Review of the Syntax of Pronouns

Recall our previous observations regarding the syntax of pronouns.

Prenominal Pronouns. Like determiners, pronouns can appear in a position that precedes a noun and any attributive adjectives.

(15)

(14) a. **the** smart students

a. **you** smart students

b. **these** stubborn linguists

b. **we** stubborn linguists

c. **those** folks

c. %them folks

This observation suggests that determiners and pronouns are syntactically equivalent and, moreover, distinct from Ns.

Complementarity. While determiners and pronouns and have an overlapping syntactic distribution, they are in complementary distribution.

(16) a. *the you smart students

(17) a.

a. *you the smart students

b. *these we stubborn linguists

b. *we these stubborn linguists

c. *those them folks

c. *them those folks

This observation suggests that determiners and pronouns are in fact two realizations of the same syntactic category.

Taken together, facts like those above suggest that pronouns and determiners are a specific realization of a broader category D(eterminer). This is an influential idea that has become the standard among syntacticians.

Consider now how this affects the kind of reasoning about the type of syntactic object that pronouns are:

- If pronouns represent projections of the category D, and
- if pronouns "replace" nominal constituents, then
- it stands to reason that nominal constituents are the projection of a D, namely a DP.

This leaves us with structures like those below, in which nominal constituents and pronouns are DP projections.

(18)

a.

DP
D NP
that |
N
professor

b.

b. DP | D

he

2.2 The Semantic Puzzle of Pronouns

Our concern now turns to how our semantic system can compute the meaning of **pronouns**. For now, we will restrict ourselves to third-person singular pronouns:

(19)
$$[\![he]\!] = ??$$
 (20) $[\![she]\!] = ??$ (21) $[\![they]\!] = ??$

Part of this question involves determining exactly what are our semantic system should yield as the extension of a pronoun and how.

Toward answering these questions, let's consider a few key properties of how pronouns contribute to the meaning of an utterance.

Pronouns Denote Entities. Given sentence like those in (22) and (23) below, it seems that the extension of a pronoun is an entity. That is, $[\![PRO]\!] \in D_e$.

(22) Context: The speaker is pointing at Kuniko.

i.
$$[\![\![\mathbf{She} \text{ sings}]\!] = T \text{ iff}$$
 Kuniko sings (by FA)

ii. $[\![\![\text{ sings}]\!] ([\![\![\mathbf{she}]\!]) = T \text{ iff}$ Kuniko sings

$$[\![\![\text{ she}]\!] = \text{Kuniko}$$

(23) Context: The speaker is pointing at Sam.

i.
$$[\![\mathbf{He} \text{ golfs}]\!] = T \text{ iff}$$
 Sam golfs (by FA)

ii. $[\![\text{golfs}]\!] ([\![\mathbf{he}]\!]) = T \text{ iff}$ Sam golfs

 $[\![\mathbf{he}]\!] = Sam$

Pronoun Extensions Vary. Comparing sentences like those in (23) and (24), we find that the extension of a pronoun varies across different kinds of contexts.

(24) Context: The speaker is pointing at Mike.

i.
$$[\![\mathbf{He} \text{ golfs}]\!] = T \text{ iff}$$
 Mike golfs (by FA)
ii. $[\![\text{ golfs}]\!] ([\![\mathbf{he}]\!]) = T \text{ iff}$ Mike golfs $[\![\mathbf{he}]\!] = \mathbf{Mike}$

Pronoun Extensions are Numerous. Comparing sentence like those in (23)–(25), we find that a pronoun can refer to many different entities in D_e .

(25) Context: The speaker is pointing at Jason.

i.
$$\llbracket \mathbf{He} \text{ golfs} \rrbracket = T \text{ iff}$$
 Jason golfs (by FA)

ii. $\llbracket \text{ golfs} \rrbracket (\llbracket \mathbf{he} \rrbracket) = T \text{ iff}$ Jason golfs

 $\llbracket \text{ he} \rrbracket = \text{Jason}$

These facts collectively pose a pretty serious problem for our semantic system as it is currently structured.

- We need to write a lexical entry for [he]—as well as for [she] and [they]—which reflects the fact that it has numerous possible values that vary depending on the context.
- It would be impossible to simply write out every possible denotation of a pronoun (i.e., $[\![he]\!] = Mike$ or Sam or Jason or ...) because D_e is theoretically infinitely large.
- Moreover, this would fail to capture the crucial fact that what [he] can denote depends on the specific context in which it is used.

Put simply, we have found here that pronouns do not have fixed denotations; they're interpretation varies across contexts. This has lead semanticists and philosophers to the conclusion that pronouns are actually interpreted like **variables**.

This is in contrast to common nouns, proper names, verbs, and other elements which have lexically fixed denotations. These are accordingly referred to as **constants**.

Our question, therefore, becomes how we can augment our semantic system in order to interpret pronouns as **context-dependent** variables.

3 Interpreting Pronouns: A First Attempt

3.1 The Basic Idea: Variable Assignments

An influential way of formalizing the meaning of pronouns that overcomes the challenges above involves augmenting our semantic notation to reflect the fact that the extension of a pronoun is computed relative to some particular context.

The typical notation that is used to represent the aspects of the context that affect the interpretation of an expression involve superscripted variables attached to the interpretation function:

(26) Interpretation relative to a context C

 $[\![X]\!]^C$ = the extension of X relative to a context C

The contextual variable *C* may be decomposed into more fine-grained parameters including conversational participants, a time, and other details of the situation of utterance.

It is common, when one is unconcerned with or abstracting away from these contextual parameter to simply use *C* or, as we have been doing, omit this variable altogether.

Our focus here is on that contextual parameter that concerns the interpretation of pronouns. This is standardly represented with the superscript *g*, which represents the interpretation of an expression relative to some **variable assignment**.

(27) Interpretation relative to variable assignment g

 $[X]^g$ = the extension of X relative to the variable assignment g

The way to think about a variable assignment is as a set of instructions for assigning a pronoun its denotation. For now, we will take variable assignments to be constituted of single specific entities taken from D_e .

As a simplified point of illustration, consider the following examples. They show how we can talk about the interpretation of the pronoun $[\![he]\!]$ with respect to different variable assignments.

(28) Referring to an extension relative to a variable assignment

- a. $[\![he]\!]^{Sam} = the extension of he in a context where a pronoun denotes Sam$
- b. $\|$ he $\|$ hike = the extension of he in a context where a pronoun denotes Mike
- c. $[\![he]\!]^{Jason}$ = the extension of *he* in a context where a pronoun denotes *Jason*

In order to ensure that the interpretation of a pronoun yields the single specific entity in D_e that constitutes the variable assignment, we can adopt the following rule of semantic interpretation:

(29) **Pronouns Rule (PR)** (preliminary)

If X is a pronoun, then, for any variable assignment g, $[\![X]\!]^g = g$.

When applied to the examples above, this rule will deliver the following type e denotations for the pronoun he as a function of the context in which it is used:

(30) Illustration of the preliminary Pronouns Rule

- a. $\llbracket \text{ he } \rrbracket^{\text{Sam}} = \text{Sam}$
- b. $\| he \|^{Mike} = Mike$
- c. $\| he \|^{Jason} = Jason$

3.2 Adjusting The Other Rules of Interpretation

The introduction of variable assignments to the interpretation function requires that we make the same adjustments to our other rules of interpretation.

Namely, we need to make sure that each rule is capable of computing the meaning of both our "usual" assignment-independent expressions and these new assignment-dependent expressions with pronouns.

We can satisfy both of these requirements by specifying each of our rules to compute **assignment-dependent** meanings from the assignment-dependent meanings of their composite parts.

(31) Functional Application (FA)

If X is a node that has two daughters, Y and Z, then, **for any variable assignment** g, if $[\![Y]\!]^g$ is a function whose domain contains $[\![Z]\!]^g$, then $[\![X]\!]^g = [\![Y]\!]^g ([\![Z]\!]^g)$.

(32) **Predicate Modification (PM)**

If X is a node that has two daughters, Y and Z, then, **for any variable assignment** g, if $[\![Y]\!]^g$ and $[\![Z]\!]^g$ are in $D_{\langle e,t\rangle}$, then $[\![X]\!]^g = [\lambda x : x \in D_e$. $[\![Y]\!]^g(x) = T$ and $[\![Z]\!]^g(x) = T$

(33) Non-Branching Nodes (NN) Rule

If X is a non-branching node that has Y as its daughter, then, **for any variable assignment g**, $\|X\|^g = \|Y\|^g$

(34) Terminal Nodes (TN) Rule

If X is a terminal node, then, **for any variable assignment g**, $[X]^g$ is specified in the lexicon.

(35) **Pronouns Rule (PR)** (preliminary)

If X is a pronoun, then, for any variable assignment g, $[X]^g = g$.

With these same rules, it will also be possible to compute **assignment-independent** interpretations.

This is ultimately a consequence of the fact that the interpretation of any **constant** by these rules, under any arbitrary variable assignment, will be equivalent to its **assignment-independent** interpretation.

- (36) a. $[Sam]^g = [Sam] = Sam$
 - b. $\| \operatorname{sings} \|^g = \| \operatorname{sings} \| = [\lambda x_e \cdot x \operatorname{golfs}]$

This is a property of our system that can we can capture in the following way:

(37) General assignment-independence of constants

If Y is a constant, then, for any variable assignment g, $[Y]^g = [Y]$.

As we will see immediately below, these notation changes will allow our semantic system to meet the goals that we laid out above. This system is able to:

- i. calculate the truth conditions for sentences containing pronouns relative to a context, and
- ii. capture the fact that the truth conditions of sentences containing a pronoun will vary along with the context t in which it is used.

3.3 Computing Truth Conditions

Let us see how these pieces come together to give us truth conditional statements like in (38).

(38) Context: The speaker is pointing at Sam

"He golfs" is T iff Sam golfs

- (39) Calculation of the Truth Conditions in (38)
 - i. "He golfs" is *T iff* (by syntax)
 - ii. "S" is T iff (by notation) DP VP
 - DP VP | V

he golfs

iii.
$$[\![S \!]\!]^{Sam} = T$$

iv. $Calculation of [\![VP \!]\!]^{Sam}$

a. $[\![VP \!]\!]^{Sam} = (by NN, TN)$

b. $[\![\lambda x_e \cdot x \text{ golfs} \!]\!]$

v. $Calculation of [\![DP \!]\!]^{Sam}$

a. $[\![DP \!]\!]^{Sam} = (by NN)$

b. $[\![he \!]\!]^{Sam} = (by PR)$

c. Sam

vii. $[\![S \!]\!]^{Sam} = T iff (by FA)$

viii. $[\![VP \!]\!]^{Sam}([\![DP \!]\!]^{Sam}) = T iff (by iv.)$

ix. $[\![\lambda x_e \cdot x \text{ golfs} \!]([\![DP \!]\!]^{Sam}) = T iff (by v.)$

x. $[\![\lambda x_e \cdot x \text{ golfs} \!]([\![DP \!]\!]^{Sam}) = T iff (by LC)$

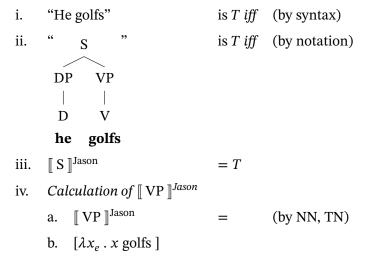
xi. Sam golfs

Thus, *He golfs*, in the context of (38), is *T iff* Sam golfs.

We can appreciate how the variable assignment contributes to the computation of the meaning of an expression, and the interpretation of a pronoun specifically, by considering a context that provides a different variable assignment:

(40) Context: The speaker is pointing at Jason "He golfs" is T iff Jason golfs

(41) Calculation of the Truth Conditions in (40)



Calculation of $[\![DP]\!]^{Jason}$ v. a. \mathbb{I} DP \mathbb{I} Jason (by NN) b. ¶ he ¶ Jason (by PR) c. Jason $[\![S]\!]^{Jason}$ vii. = T iff (by FA) $[VP]^{Jason}([DP]^{Jason})$ = T iff (by iv.) viii. $[\lambda x_e \cdot x \text{ golfs }]([\![DP]\!]^{Jason}) = T \text{ iff} \quad (by v.)$ $[\lambda x_e \cdot x \text{ golfs }](\text{Jason})$ = T iff (by LC) X. Jason golfs xi.

Thus, *He golfs*, in the context of (40), is *T iff* Jason golfs.

To summarize what we've done in this section:

- We have paired our semantic interpretation function with a superscript: $[\![\cdot]\!]^g$.
- This superscript represents particular parameters of the context in which an expression is interpreted.
- Namely, it is a **variable assignment**, which provides the instructions for interpreting a pronoun in that context.

4 Variable Assignments as Functions

4.1 A Bug in the System

Despite the benefits of this approach to pronouns, it suffers from a critical flaw.

And this is the fact that it is possible to have multiple instances of the pronoun *he* in a single sentence where each instance of the pronoun refers to a different entity:

(42) Context: The speaker points first at Sam and then at Mike. $\|$ **He** likes **him** $\|$ = T iff **Sam** likes **Mike**

If a variable assignment only resolves the meaning of a variable to a single specific entity, as our system currently does, then all pronouns in a sentence will have to be interpreted as referring to that entity:

(43) a.
$$[\![\text{He likes him }]\!]^{\text{Sam}} = T$$
 iff Sam likes Sam b. $[\![\text{He likes him }]\!]^{\text{Mike}} = T$ iff Mike likes Mike

This is another way of saying that we haven't fully captured the fact that an expression with a pronoun—and, in fact, the pronoun itself—has numerous possible interpretations.

As we see here, this is possible even within a single context.

4.2 Introducing Indices

The standard solution to this problem employs an idea that was initially introduced in the domain of syntax as a means for indicating the referentiality of pronouns.

The idea begins from the assertion that every instance of a pronoun is assigned an **index**. While this was initially done with alphabetic indices, it has become standard in the domain of semantics to use numerical indices.

(44) Syntactic representation of indexed pronominals

- a. He₁ likes him₂.
- b. He₇ likes him₉.
- c. He_2 likes him_2 .

The intention was that the semantics would be able to interpret indices as pointers to the reference of a pronoun. The effects that were intended are as follows:

• Pronouns that are assigned the same index will necessarily refer to the same entity.

(45) Semantic interpretation of co-indexed pronominals

 $[He_2 \text{ likes him}_2]^g = T \text{ iff Mike likes Mike}$

• Pronouns that are assigned different indices will necessarily refer to different entities.

(46) Semantic interpretation of contra-indexed pronominals

 $[\![He_1]\!]$ He₁ likes him₂ $[\![g]\!]$ = T iff Sam likes Mike

As stated, this is somewhat vague and wishful. The problem we face is how exactly we can formalize this idea within our semantic system.

4.3 Variable Assignment Functions

In order to move forward, we have to give up the idea that a variable assignment is constituted of a single specific entity.

Instead, the critical innovation is that we should conceive of a variable assignment as a function from indices (natural numbers) to the set of individuals.

That is, pronouns are interpreted by reference to a **variable assignment function**:

(47) Representation of possible variable assignment functions

- a. $g = \{ \langle 1, Sam \rangle \}$
- b. $h = \{ \langle 1, Sam \rangle, \langle 2, Mike \rangle \}$
- c. $j = \{ \langle 7, Jason \rangle, \langle 9, Cedric \rangle, \langle 53, Marcel \rangle, ... \}$

To put things in different terms:

- The earlier concept of variable assignments had our interpretation function map a pronoun directly to the entity under discussion.
- Our new concept of variable assignment function can be thought of as essentially doing essentially the same thing; it maps a pronoun from its index to the appropriate member of the set of entities under discussion.
- As a model of our linguistic competence, we might consider indices as pointers to the "memory slot" that an entity is assigned to.

Implementing this new technology into our semantic system is going to require that we adjust the Pronoun Rule.

More specifically, we will need the Pronoun Rule, upon identifying an indexed pronoun, to apply the variable assignment function to the index and return the appropriate entity as a value.

This is something that we can accomplish with the following version of the rule:

= (by PR)

(48)**Pronouns Rule (PR)**

i.

a.

If X is a pronoun bearing an index n, then, for any variable assignment g, if n is in the domain of g, then $[X]^g = g(n)$.

The work that this rule will do upon identifying an indexed pronoun is illustrated in the following examples.

(49)Illustration of the Pronouns Rule $[\![he_1]\!]^g$

```
ii.
           g(1)
                           = (by definition of g)
     iii.
           Sam
b.
     i.
            \llbracket \text{he}_2 \rrbracket^h
                           = (by PR)
     ii.
           h(2)
                           = (by definition of h)
           Mike
     iii.
c.
     i.
            [\![ he_{53} ]\!]^j
                                (by PR)
     ii.
           i(53)
                           = (by definition of i)
     iii.
           Marcel
```

d. i.
$$\llbracket he_7 \rrbracket^g = (by PR)$$

ii. $g(7) = (by definition of g)$
iii. undefined

4.4 Computing Truth Conditions

Let us see how these pieces come together to give us truth conditional statements like in (50).

- (50) Context: The speaker first points at Sam then at Mike Variable Assignment: $g = \{ \langle 1, Sam \rangle, \langle 2, Mike \rangle \}$ "He likes him" is T iff Sam likes Mike
- (51) Calculation of the Truth Conditions
 - i. "He likes him" is T iff (by syntax)
 ii. "S" is T iff (by notation) $DP_1 VP$ $DP_1 V DP_2$ $he_1 likes$ D_2 him_2
 - iii. $\llbracket S \rrbracket^g = T$
 - iv. $Calculation of [\![DP_1]\!]^g$
 - a. $[DP_1]^g$ = (by NN)
 - b. $[he_1]^g = (by PR)$
 - c. g(1) = (by def. of g)
 - d. Sam
 - v. Calculation of $[\![DP_2]\!]^g$
 - a. $[DP_2]^g = (by NN)$
 - b. $[\![him_2]\!]^g = (by PR)$
 - c. g(2) = (by def. of g)
 - d. Mike
 - vi. Calculation of $[VP]^g$
 - a. $[VP]^g = (by FA)$
 - b. $[\![likes]\!]^g([\![DP_2]\!]^g) = (by TN)$
 - c. $[\lambda x_e . [\lambda y_e . y \text{ likes } x]]([\![DP_2]\!]^g) = (by v.)$
 - d. $[\lambda x_e . [\lambda y_e . y \text{ likes } x]]$ (Mike) = (by LC)
 - e. $[\lambda y_e \cdot y \text{ likes Mike }]$

```
vii. [\![ S ]\!]^g = T iff (by FA)

viii. [\![ VP ]\!]^g ([\![ DP_1 ]\!]^g) = T iff (by vi.)

ix. [\![ \lambda y_e . y ]\!] ikes Mike [\![ (\![ DP_1 ]\!]^g)] = T iff (by iv.)

ix. [\![ \lambda y_e . y ]\!] ikes Mike [\![ (Sam)]] = T iff (by LC)

xi. Sam likes Mike
```

Thus, *He likes him*, in the context of (50), is *T iff* Sam likes Mike.

Let us summarize what we have done in this lecture:

- There are two ingredients for modeling the **context-dependance** of the meaning of pronouns:
 - i. The interpretation function is parameterized for aspects of the context that affect the interpretation of an expression. This specifically includes the the variable assignment:
 - (52) $[\cdot]^g$
 - ii. The Pronouns Rule (PR) uses this contextual parameter (i.e., the variable assignment function) to link the pronoun to its extension:
 - (53) **Pronouns Rule (PR)**

If X is a pronoun bearing an index n, then, for any variable assignment g, if n is in the domain of g, then $[X]^g = g(n)$.

- There are two ingredients for determining the extension that is assigned to a pronoun:
 - i. The pronoun will carry a numerical index:
 - (54) PRO_n
 - ii. The contextually-determined variable assignment function maps that index to an entity:

A question we will ask is what role the other properties of pronouns play in their interpretation. For instance, given that it is appropriate to refer to Sam as *he*, we will need to avoid the result below:

(56) Context: The speaker points at Sam Variable Assignment:
$$g = \{\langle 1, Sam \rangle, ... \}$$

i. $[\![her_1]\!]^g = (by PR)$
ii. $g(1) = (by definition of g)$
iii. Sam

In other words, we will need to find a way to restrict the pronoun *she* to referring to female entities, and similarly for masculine *he* and non-binary *they*.

5 Practice

Please show how our semantic system could compute the truth-conditional statement below:

