Quantificational DPs

Lecture 14

March 12, 2024

Announcements:

This lecture is supplemented by the following readings:

- Heim & Kratzer: Ch.6 (131–138)

Your fifth homework assignment is available and is due on March 21st.

1 Introduction

We currently have a semantic system for interpreting sentences that relies on the following set of rules:

(1) **Functional Application (FA)**
If X is a node that has two daughters, Y and Z, and if $[\![\text{Y}]\!]$ is a function whose domain contains $[\![\text{Z}]\!]$, then $[\![\text{X}]\!] = [\![\text{Y}]\!]([\![\text{Z}]\!])$.

(2) **Predicate Modification (PM)**
If X is a node that has two daughters, Y and Z, and if $[\![\text{Y}]\!]$ and $[\![\text{Z}]\!]$ are in $D_{\langle e,t\rangle}$, then $[\![\text{X}]\!] = [\lambda x : x \in D_e . [\![Y]\!](x) = T \text{ and } [\![Z]\!](x) = T\ ]$

(3) **Non-Branching Nodes (NN) Rule**
If X is a non-branching node that has Y as its daughter, then $[\![\text{X}]\!] = [\![\text{Y}]\!]$

(4) **Terminal Nodes (TN) Rule**
If X is a terminal node, then $[\![\text{X}]\!]$ is specified in the lexicon.

These rules are able to interpret structures built from lexical entries like the following:

(5) **Type *e* expressions**
$[\![\text{NAME}]\!]$ = the thing referred to with NAME
$[\![\text{the NP}]\!]$ = the unique x such that $[\![\text{NP}]\!](x) = T$

(6) **Type $\langle e, e\rangle$ expressions**
$[\![\text{D}_{\varnothing}]\!] = [\lambda x : x \in D_e . x]$

(7) **Type $\langle\langle e,t\rangle,e\rangle$ expressions**
$[\![\text{ the }]\!] = [\lambda f : f \in D_{\langle e,t\rangle}$ and there is a unique $x \in C$ such that $f(x) = T$.
the unique x such that $f(x) = T]$

(8) **Type $\langle e,t\rangle$ expressions**

a. $[\![\text{ VERB}_{intrans}]\!] = [\lambda x : x \in D_e . x$ VERBs]
b. $[\![\text{ ADJ}_{int}]\!] = [\lambda x : x \in D_e . x$ is ADJECTIVE]
c. $[\![\text{ NOUN }]\!] = [\lambda x : x \in D_e . x$ is a NOUN]

(9) **Type $\langle e,\langle e,t\rangle\rangle$ expressions**
$[\![\text{ VERB}_{trans}]\!] = [\lambda x : x \in D_e . [\lambda y : y \in D_e . y$ VERBs $x]$

(10) **Type $\langle e,\langle e,\langle e,t\rangle\rangle\rangle$ expressions**
$[\![\text{ VERB}_{ditrans}]\!] = [\lambda x : x \in D_e . [\lambda y : y \in D_e . [\lambda z : z \in D_e . z$ VERBs x $y]]]$

(11) **Type $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$ expressions**

a. $[\![\text{ is }]\!] = [\ \lambda f : f \in D_{\langle e,t\rangle} . f]$
b. $[\![\text{ a }]\!] = [\ \lambda f : f \in D_{\langle e,t\rangle} . f]$
c. $[\![\text{ ADJ}_{sub}]\!] = [\lambda f : f \in D_{\langle e,t\rangle} . [\lambda x : x \in D_e . f(x) = T$ and x is ADJ for entities in $\{y : f(y) = T\}]]$

Our attention has shifted recently to understanding how our semantic system might be able to interpret common nouns in the argument positions of predicates. In our most recent meetings, we have focused on providing a semantics for definite descriptions and the definite article *the*. These results are summarized above.

(12) Felix likes **the professor**.

Today we turn to the semantics of **quantificational DPs**, a term which refers to DPs with **quantificational determiners**, also simply called **quantifiers**. Some relevant examples are provided below:

(13) a. **A/some professor** golfs.
b. **No professor** golfs.
c. **Every professor** golfs.
d. **Few professors** golfs.
e. **Most professors** golfs.
f. **Three professors** golfs.

Our goal for today is to develop an understanding of the semantic type and extension of **quantificational DPs** as a whole.

In particular, we will see some challenges to the simple hypothesis that they are type *e* expressions that denote entities. From this we will motivate the alternative hypothesis that they are better treated as expressions of type $\langle\langle e,t\rangle,t\rangle$.

This will allow us to spend our next meeting working out the semantics of **quantificational determiners** that deliver these result and as well as some of their other properties.

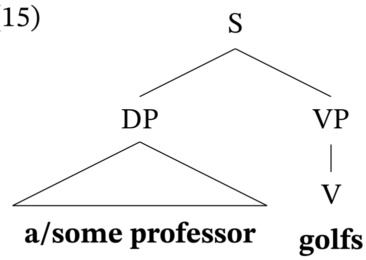
2 The Semantic Type of Quantificational DPs: First Attempt

Our goal is to develop our semantic system to interpret sentences with a quantificational DP (Q-DP):

(14) **A/some/no/every professor** golfs.

Toward this end, we can start by determining what semantic type a Q-DP must have. For this we can assume that sentences with Q-DPs have syntactic structures like those below:

(15)



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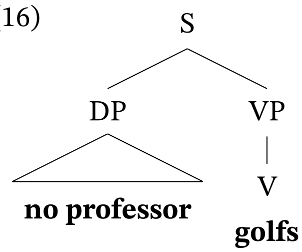

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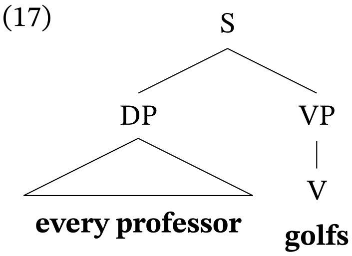


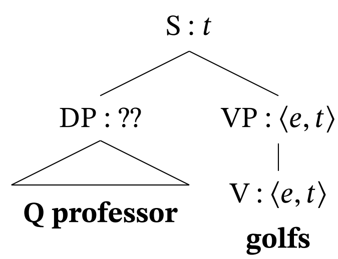
1. Known Semantic Types. We start by listing out the semantic types we know and annotating the syntactic representation with this information.

(18) **Known Semantic Types in (19)**

a. $[\![\text{S}]\!] \in D_t$

b. $[\![\text{VP}]\!] \in D_{\langle e,t \rangle}$

(19)



2. Reasoning out the Semantic Type of a Q-DP. We can appeal to the known semantic types and the rules of composition, to determine that a Q-DP could be of type *e*.

- The S node fits the structural description for Functional Application. So, the extension of the DP must combine with the extension of the VP to return the extension of the S.

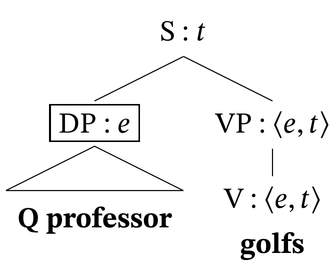
  (20) $[\![\text{DP}]\!] + [\![\text{VP}]\!] = [\![\text{S}]\!]$

- Because $[\![\text{VP}]\!]$ is a function of type $\langle e, t \rangle$ and $[\![\text{S}]\!]$ is of type t, the extension of the VP could take the extension of the DP as its argument to return the type t extension of the S as its value.

  (21) $[\![\text{VP}]\!]([\![\text{DP}]\!]) = [\![\text{S}]\!]$

- Thus, a quantificational DP could be of type *e*.

  (22)



3 Against a Type *e* Extension for Quantificational DPs

Despite being the obvious conclusion, there is a significant amount of evidence that it is incorrect.

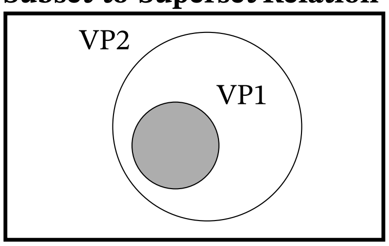
Namely, Q-DPs do not display the same kind of logical behavior that can be observed for both proper names and definite descriptions. This suggests that they in fact have a different semantics and, therefore, a different semantic type.

It is worth pointing out that these observations that follow not only suggest that Q-DPs are distinct, but they validate our assumptions that proper names and definite description can be treated the same.

3.1 Subset-to-Superset Inferences

Subset-to-superset inferences are licensed when the truth of one proposition entails the truth of another proposition.

(23) **Subset-to-Superset Relation**



Suppose we have two VPs such that $[\![\text{VP1}]\!](x_e) = T$ entails that $[\![\text{VP2}]\!](x_e) = T$. It follows from our semantics system that inferences of the following type are valid:

(24) **Subset-to-Superset Inference**
IF $[\![\text{VP1}]\!](x_e) = T$
THEN $[\![\text{VP2}]\!](x_e) = T$

Applied to Entities. As the examples below show, when two such VPs are applied to an entity, the result is a valid subset-to-superset inference:

(25) **A valid inference**
IF ⟦ **Sam** golfs well ⟧ = T
THEN ⟦ **Sam** golfs ⟧ = T

(26) **A valid inference**
IF ⟦ **The professor** golfs well ⟧ = T
THEN ⟦ **The professor** golfs ⟧ = T

Intuitively, if some entity golfs well, then it is necessarily true that that entity golfs.

Applied to Q-DPs. But, when applied to a Q-DP, the result is not necessarily a valid inference:

(27) **An invalid inference**
IF ⟦ **Few professors** golf well ⟧ = T
THEN ⟦ **Few professors** golf ⟧ = T

(28) **An invalid inference**
IF ⟦ **No professor** golfs well ⟧ = T
THEN ⟦ **No professor** golfs ⟧ = T

In neither case is the expressed inference a valide one. It is possible that either few or no professors golf well while, at the same time, possibly many professors golf.

It is due to this difference in entailment relationships that we also observe the following contrast:

(29) **A non-contradiction**
⟦ **No professor** golfs well ⟧ = T AND
⟦ **No professor** golfs ⟧ = F

(30) **A contradiction**
⟦ **Sam** golfs well ⟧ = T AND
⟦ **Sam** golfs ⟧ = F

It can be true that no professor golfs well while simultaneously being false that no professor golfs. The same cannot hold for an entity.

Observations like this suggests that proper names and definite descriptions are both of type e. But Q-DPs are not of type e and sentences containing them have a different semantics.

3.2 The Law of Contradiction

The **Law of Contradiction** states that a proposition p cannot be both true and false at the same time.

(31) **The Law of Contradiction**
not (p and not p)

Suppose we have some VP such that ⟦ VP ⟧$(x_e) = T$. This entails that ⟦ not VP ⟧$(x_e) = F$. Thus, it follows from our semantic system that statements of the following type are contradictory:

(32) **A contradiction**
⟦ VP ⟧$(x_e) = T$ AND
⟦ not VP ⟧$(x_e) = T$

Applied to Entities. When applied to DPs that denote entities, the following statements are contradictions:

(33) **A contradiction**
⟦ **Sam** golfs ⟧ = T AND
⟦ **Sam** doesn't golf ⟧ = T

(34) **A contradiction**
⟦ **The professor** golfs ⟧ = T AND
⟦ **The professor** doesn't golf ⟧ = T

The proposition that some entity golfs is contradicted by the proposition that that entity does not golf.

Applied to Q-DPs. But, when applied to a proposition that contains a Q-DP, the result is not necessarily a set of contradictory statements:

(35) **A non-contradiction**
⟦ **Three professors** golf ⟧ = T AND
⟦ **Three professors** don't golf ⟧ = T

(36) **A non-contradiction**
⟦ **Some professor** golfs ⟧ = T AND
⟦ **Some professor** doesn't golf ⟧ = T

These statements don't give rise to a contradiction in contexts where we are referring to different sets of professors. It is possible for one or more to golf while others don't golf.

Observations like this suggests that proper names and definite descriptions are both of type e. But Q-DPs are not of type e and sentences containing them have a different semantics.

3.3 The Law of Excluded Middle

The **Law of Excluded Middle** states that either a proposition is true or it is false.

(37) **The Law of Excluded Middle**
either p or not p

Suppose we have some VP such that $[\![\text{ VP }]\!](x_e)$. It is necessarily the case that either $[\![\text{ VP }]\!](x_e) = T$ or that $[\![\text{ not VP }]\!](x_e) = F$. Thus, it follows from our semantic system that statements of the following type are necessarily true:

(38) EITHER $[\![\text{ VP }]\!](x_e) = T$
OR $[\![\text{ not VP }]\!](x_e) = T$

Applied to Entities. The examples below show that, when applied to DPs that denote entities, the resulting statements are necessarily true:

(39) **A logical truth**
EITHER ⟦ **Sam** golfs ⟧ = T
OR ⟦ **Sam** doesn't golf ⟧ = T

(40) **A logical truth**
EITHER ⟦ **The professor** golfs ⟧ = T
OR ⟦ **The professor** doesn't golf ⟧ = T

Intuitively, statements of this kind exhaust the logical possibility space.

Applied to Q-DPs. But, when applied to Q-DPs, the resulting statement is not necessarily true proposition:

(41) **Not a logical truth**
EITHER ⟦ **Most professors** golf ⟧ = T
OR ⟦ **Most professor** don't golf ⟧ = T

(42) **Not a logical truth**
EITHER ⟦ **Every professor** golfs ⟧ = T
OR ⟦ **Every professor** doesn't golf ⟧ = T

It is a logical possibility that exactly half of the professors golf or that only some of them golf, in which case neither statement would be true.

Observations like this suggests that proper names and definite descriptions are both of type e. But Q-DPs are not of type e and sentences containing them have a different semantics.

3.4 Interim Summary

The state of affairs outlined above, for a long time, presented a fairly difficult puzzle for those interested in the study of natural language semantics.

Modern logicians and semanticists, however, agree that these facts collectively point to the conclusion that quantificational DPs simply do not denote entities and are, therefore, not expressions of type e.

Of course, this raises the question of what semantic type they do have. In what follows we will motivate the idea that Q-DPs are expression of type $\langle\langle e,t\rangle,t\rangle$:

(43) ⟦ **a/some/no/every NP** ⟧ $\in D_{\langle\langle e,t\rangle,t\rangle}$

4 The Semantic Type of Quantificational DPs: Second Attempt

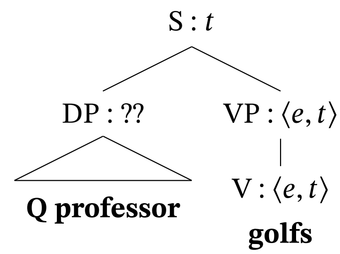
1. Known Semantic Types. We start by listing out the semantic types we know and annotating the syntactic representation with this information.

(44) **Known Semantic Types in (45)**

a. $[\![\text{S}]\!] \in D_t$

b. $[\![\text{VP}]\!] \in D_{\langle e,t\rangle}$

(45)



2. Reasoning out the Semantic Type of a Q-DP. We can appeal to the known semantic types and the rules of composition, to determine that a Q-DP must be of type $\langle\langle e,t\rangle,t\rangle$.

- The S node fits the structural description for Functional Application. So, the extension of the DP must combine with the extension of the VP to return the extension of the S.

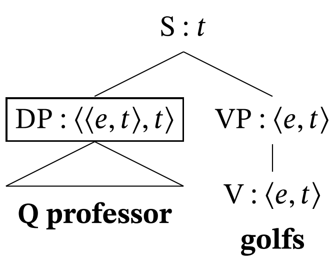
  (46) $[\![\text{DP}]\!] + [\![\text{VP}]\!] = [\![\text{S}]\!]$

- We determined in section 3 that treating Q-DPs as type e arguments of the type $\langle e,t\rangle$ $[\![\text{VP}]\!]$ does not deliver the correct results in several instances.
- So, because $[\![\text{VP}]\!]$ is a function of type $\langle e,t\rangle$ and $[\![\text{S}]\!]$ is of type t, the extension of the DP must take as its argument the extension of the VP to return the type t extension of the S as its value.

  (47) $[\![\text{DP}]\!]([\![\text{VP}]\!]) = [\![\text{S}]\!]$

- Thus, a quantificational DP must be of type $\langle\langle e,t\rangle,t\rangle$, whereas proper names and definite descriptions denote expressions of type e.

(48)



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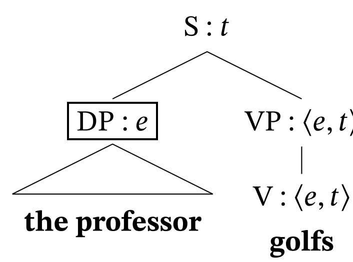

5 The Extension of Quantificational DPs

We now know what kind of expression a quantificational DP must be. It is a type $\langle\langle e,t\rangle,t\rangle$ function, meaning it maps **functions** from entities to truth values to **truth values**.

We therefore need to develop a lexical entry for *the* that does the following:

- assigns as its extension a function of type $\langle\langle e,t\rangle,t\rangle$ and
- allows our system to derive an appropriate truth-conditional statement for sentences with Q-DPs.

Reasoning out the Extension of Q-DPs. We turn now to reasoning out a type $\langle\langle e,t\rangle,t\rangle$ extension for quantificational DPs.

- Let us start by considering simple sentences with quantificational DPs like those below:

(50) a. **A/some professor** golfs.
b. **No professor** golfs.
c. **Every professor** golfs.

- A key guiding intuition is that Q-DPs are taking the type $\langle e,t\rangle$ predicate as their argument and returning a truth value if the predicate satisfies some condition:

(51) **some professor** says that its $[\![$ VP $]\!]$ argument returns T of some professor.
a. $[\![$ some professor $]\!]([\![$ VP $]\!]) = T$ *iff* there is some professor x such that $[\![$ VP $]\!](x) = T$
b. $[\![$ some professor $]\!]([\![$ golfs $]\!]) = T$ *iff* there is some professor x s.t. $[\![$ golfs $]\!](x) = T$

(52) **no professor** says that its $[\![$ VP $]\!]$ argument returns T of no professor.
a. $[\![$ no professor $]\!]([\![$ VP $]\!]) = T$ *iff* there is no professor x such that $[\![$ VP $]\!](x) = T$
b. $[\![$ no professor $]\!]([\![$ golfs $]\!]) = T$ *iff* there is no professor x such that $[\![$ golfs $]\!](x) = T$

(53) **every professor** says that its $[\![$ VP $]\!]$ argument returns T of every professor.
a. $[\![$ every professor $]\!]([\![$ VP $]\!]) = T$ *iff* for all x, if x is a professor, then $[\![$ VP $]\!](x) = T$
b. $[\![$ every professor $]\!]([\![$ golfs $]\!]) = T$ *iff* for all x, if x is a professor, then $[\![$ golfs $]\!](x) = T$

- With this in mind, it is possible to provide lexical entries for Q-DPs that take a type $\langle e,t\rangle$ VP as their argument and returns T *iff* the appropriate conditions are met:

(54) $[\![$ a/some professor $]\!] = [\lambda f : f \in D_{\langle e,t\rangle}$. there is some professor x such that $f(x) = T]$

(55) $[\![$ no professor $]\!] = [\lambda f : f \in D_{\langle e,t\rangle}$. there is no professor x such that $f(x) = T]$

(56) $[\![$ every professor $]\!] = [\lambda f : f \in D_{\langle e,t\rangle}$. for all x, if x is a professor, then $f(x) = T]$

Calculating the Truth Conditions. Let's see how our extension for quantificational DPs contributes to the truth conditions of a sentence.

(57) **Calculation of the truth conditions of *Some professor golfs***

i.	$[\![\ [_S$ Some professor $[_{VP}$ golfs $]]\ ]\!]$	$= T$ *iff*	(by FA, TN)
ii.	$[\![$ some professor $]\!]([\![$ golfs $]\!])$	$= T$ *iff*	(by (54))
iii.	$[\lambda f : f \in D_{\langle e,t\rangle}$. there is some professor x such that $f(x) = T]([\![$ golfs $]\!])$	$= T$ *iff*	(by LC)
iv.	there is some professor x such that $[\![$ golfs $]\!](x) = T$	$= T$ *iff*	(by LE)
v.	there is some professor x such that $[\lambda y \in D_e$. y golfs$](x) = T$	$= T$ *iff*	(by LC, def.)
vi.	there is some professor x such that x golfs		

(58) **Calculation of the truth conditions of *No professor golfs***

i.	$[\![\ [_S$ No professor $[_{VP}$ golfs $]]\ ]\!]$	$= T$ *iff*	(by FA, TN)
ii.	$[\![$ no professor $]\!]([\![$ golfs $]\!])$	$= T$ *iff*	(by (55))
iii.	$[\lambda f : f \in D_{\langle e,t\rangle}$. there is no professor x such that $f(x) = T]([\![$ golfs $]\!])$	$= T$ *iff*	(by LC)
iv.	there is no professor x such that $[\![$ golfs $]\!](x) = T$	$= T$ *iff*	(by LE)
v.	there is no professor x such that $[\lambda y \in D_e . y$ golfs$](x) = T$	$= T$ *iff*	(by LC, def.)
vi.	there is no professor x such that x golfs		

(59) **Calculation of the truth conditions of *Every professor golfs***

i.	$[\![\ [_S$ Every professor $[_{VP}$ golfs $]]\ ]\!]$	$= T$ *iff*	(by FA, TN)
ii.	$[\![$ every professor $]\!]([\![$ golfs $]\!])$	$= T$ *iff*	(by (56))
iii.	$[\lambda f : f \in D_{\langle e,t\rangle}$. for all x, if x is a professor, then $f(x) = T]([\![$ golfs $]\!])$	$= T$ *iff*	(by LC)
iv.	for all x, if x is a professor, then $[\![$ golfs $]\!](x) = T$	$= T$ *iff*	(by LE)
v.	for all x, if x is a professor, then $[\lambda y \in D_e . y$ golfs$](x) = T$	$= T$ *iff*	(by LC, def.)
vi.	for all x, if x is a professor, then x golfs		

6 In Favor of a Type $\langle\langle e,t\rangle, t\rangle$ Extension for Quantificational DPs

The lexical entries proposed above work and they deliver an intuitively accurate set of truth conditions.

But it is worth taking some time to ask if these are the empirically correct truth conditions. That is, we should ask whether this solution to the semantics of quantificational DPs avoids those problems that we pointed out for the type e hypothesis in section 3.

6.1 Subset-to-Superset Inferences

Contrary to the predictions of a type e analysis for Q-DPs, the following is not a valid inference:

(60) **An invalid inference**
IF $[\![$ **No professor** golfs well $]\!] = T$
THEN $[\![$ **No professor** golfs $]\!] = T$

Consider the predicted truth conditions for each of comopnent sentences, which are as follows:

(61) $[\![$ No professor golfs well $]\!] = T$ *iff* there is no professor x such that x golfs well

(62) $[\![$ No professor golfs $]\!] = T$ *iff* there is no professor x such that x golfs

Indeed the truth conditions of (61) do not entail the truth conditions of (62). Just because no professor golfs well does not mean that no professor golfs. Thus, we correctly predict that the inference is invalid.

It is for the same reason that we also predict that the following statement does not give rise to a contradiction:

(63) **A non-contradiction**
⟦ **No professor** golfs well ⟧ = T AND
⟦ **No professor** golfs ⟧ = F

All that's required is that every professor that does golf simply does not golf well.

6.2 Law of (Non-)Contradiction

Contrary to the predictions of a type e analysis for Q-DPs, the following statement does not give rise to a contradiction:

(64) **A non-contradiction**
⟦ **Some professor** golfs ⟧ = T AND
⟦ **Some professor** doesn't golf ⟧ = T

Let's again consider the predicted truth conditions for the component sentences:

(65) ⟦ Some professor golfs ⟧ = T *iff* there is some professor x such that x golfs

(66) ⟦ Some professor doesn't golf ⟧ = T *iff* there is some professor x such that x does not golf

Given these truth conditions, each of the sentences above can be T at the same time. We only require a context in which there are two professors, such that one golfs and the other doesn't.

6.3 Law of Excluded Middle

Contrary to the predictions of a type e analysis for Q-DPs, the following statement does not represent a logical necessity:

(67) **Not a logical truth**
EITHER ⟦ **Every professor** golfs ⟧ = T
OR ⟦ **Every professor** doesn't golf ⟧ = T

Once again, we can consider the predicted truth conditions of the component sentences:

(68) ⟦ Every professor golfs ⟧ = T *iff* for all x, if x is a professor, then x golfs

(69) ⟦ Every professor doesn't ⟧ = T *iff* for all x, if x is a professor, then x doesn't golf

Given these truth conditions, (67) is not expected to be a logical truth. This statement can be F given a context in which at least one professor golfs and at least one professor doesn't.

7 Looking Ahead

On the basis of what has preceded, we have hypothesized that quantificational DPs have type $\langle\langle e,t\rangle,t\rangle$ extensions like those shown below:

(70) $[\![$ a/some professor $]\!] = [\lambda f : f \in D_{\langle e,t\rangle}$. there is some professor x such that $f(x) = T]$

(71) $[\![$ no professor $]\!] = [\lambda f : f \in D_{\langle e,t\rangle}$. there is no professor x such that $f(x) = T]$

(72) $[\![$ every professor $]\!] = [\lambda f : f \in D_{\langle e,t\rangle}$. for all x, if x is a professor, then $f(x) = T]$

Of course, these DPs are themselves compositional, meaning that these extensions must be computed as a function of the meaning of the parts and how they are combined:

(73) $[\![$ a/some/no/every $]\!] + [\![$ NP $]\!] = [\![$ DP $]\!]$

Developing our semantic system to do this will be the focus of our next lecture. Among the things that we will need to do in that meeting is determine the semantic type of quantifiers

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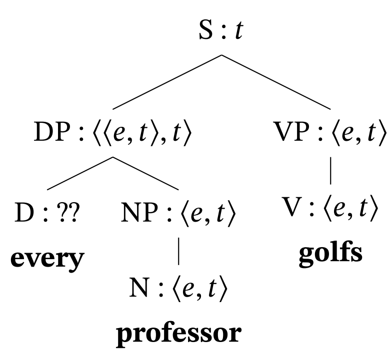
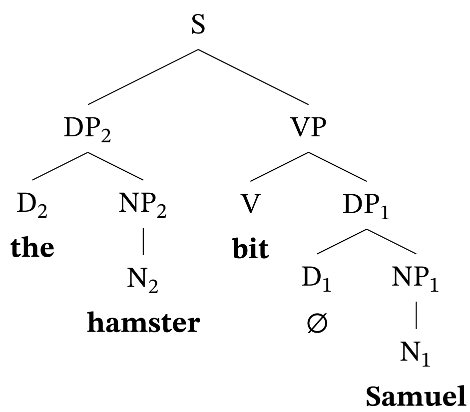


This in turn will allow us to reason out a suitable extension that delivers the appropriate truth conditions as well as some other properties of quantificational determiners.

8 Practice

Exercise. Please provide a semantic proof of the truth-conditions of the following sentence:

(75)



In order to do this, we will need to provide the extensions of the lexical items and provide a step-by-step proof of the intuitive truth conditions.