Intersective and Subsective Adjectives

Lecture 11

February 15, 2024

Announcements:

This lecture is supplemented by the following readings:

- Heim & Kratzer: Ch.4 (67–73)

Your fourth homework assignment is available and is due on February 20th.

1 Introduction

We currently have a semantic system for interpreting sentences that relies on the following set of rules:

(1) **Functional Application (FA)**
If X is a node that has two daughters, Y and Z, and if $[\![\text{Y}]\!]$ is a function whose domain contains $[\![\text{Z}]\!]$, then $[\![\text{X}]\!] = [\![\text{Y}]\!]([\![\text{Z}]\!])$.

(2) **Predicate Modification (PM)**
If X is a node that has two daughters, Y and Z, and if $[\![\text{Y}]\!]$ and $[\![\text{Z}]\!]$ are in $D_{\langle e,t\rangle}$, then $[\![\text{X}]\!] = [\lambda x : x \in D_e \,.\, [\![Y]\!](x) = T \text{ and } [\![Z]\!](x) = T\ ]$

(3) **Non-Branching Nodes (NN) Rule**
If X is a non-branching node that has Y as its daughter, then $[\![\text{X}]\!] = [\![\text{Y}]\!]$

(4) **Terminal Nodes (TN) Rule**
If X is a terminal node, then $[\![\text{X}]\!]$ is specified in the lexicon.

These rules are able to interpret structures built from lexical entries like the following:

(5) **Proper Nouns as entities**
$[\![\text{NAME}]\!]$ = the thing referred to with NAME

(6) **Type $\langle e, t\rangle$ expressions**
a. $[\![\text{VERB}_{intrans}]\!] = [\lambda x : x \in D_e \,.\, x \text{ VERBs}\ ]$
b. $[\![\text{ADJ}]\!] = [\lambda x : x \in D_e \,.\, x \text{ is ADJECTIVE}\ ]$
c. $[\![\text{NOUN}]\!] = [\lambda x : x \in D_e \,.\, x \text{ is a NOUN}\ ]$

(7) **Type $\langle e, \langle e, t \rangle\rangle$ expressions**

$[\![\text{VERB}_{trans}]\!] = [\lambda x : x \in D_e \,.\, [\lambda y : y \in D_e \,.\, y \text{ VERBs } x]$

(8) **Type $\langle\langle e, t \rangle, \langle e, t \rangle\rangle$ expressions**

a. $[\![\text{is}]\!] = [\, \lambda f : f \in D_{\langle e,t \rangle} \,.\, f]$

b. $[\![\text{a}]\!] = [\, \lambda f : f \in D_{\langle e,t \rangle} \,.\, f]$

We have recently turned our attention to the semantics of adjectives, adjectival modification, and more complex nominal expressions. During our last few lectures we focused on interpreting predicate adjectives and predicate nominals:

(9) Felix is **gray**. (10) Felix is **a cat**. (11) Felix is **a gray cat**.

During our last meeting we formulated the rule of Predicate Modification to compute truth conditional statements, like the ones below, for sentences with attributive adjectives:

(12) a. "Felix is **a gray cat**" is T *iff* Felix is **gray** and Felix is **a cat**.

b. "Sara is a **single lady**" is T *iff* Sara is **single** and Sara is a **lady**.

c. "Sunny is a **bald hamster**" is T *iff* Sunny is **bald** and Sunny is a **hamster**.

Today we are concerned with the observation that truth-conditional statements like (12) are not always accurate for describing the meaning of a certain class of adjectives:

(13) a. ?? "Richard is **a tall jockey**" is T *iff* Richard is **tall** and Richard is a **jockey**.

b. ?? "Barack is **a young ex-president**" is T *iff* Barack is **young** and Barack is **an ex-president**.

c. ?? "Proxima Centauri is **a cold star**" is T *iff* P.C. is **cold** and P.C. is a **star**.

This initial observation suggests that Predicate Modification is not the correct mode of composition for all attributive adjectives. Moreover, it suggests that adjectives come in (at least) two different flavors:

(14) **Type $\langle e, t \rangle$ Intersective Adjectives**

$[\lambda x : x \in D_e \,.\, x \text{ is ADJ}]$

(15) **Type $\langle\langle e, t \rangle, \langle e, t \rangle\rangle$ Subsective Adjectives**

$[\lambda f : f \in D_{\langle e,t \rangle} \,.\, [\lambda x : x \in D_e \,.\, f(x) = T \text{ and } x \text{ is ADJ for entities in } \{y : f(y) = T\}]]$

As we will see, lexical entries like (15) allow us to compose subsective adjectives with the nouns they modify via our familiar rule of Functional Application.

However, we will also have to contend with some complications that arise when reconsider adjectival predicates.

2 The Rule of Predicate Modification: A Review

During our last meeting, our goal was to develop our semantic system so that it is able to interpret sentences that contain attributive adjectives, like in (16).

(16)
$$\begin{array}{cc} \multicolumn{2}{c}{\boxed{\mathrm{NP}_3 : \langle e,t\rangle}} \\ \mathrm{AP} : \langle e,t\rangle & \mathrm{NP}_4 : \langle e,t\rangle \\ | & | \\ \mathrm{A} : \langle e,t\rangle & \mathrm{N}_4 : \langle e,t\rangle \\ \textbf{gray} & \textbf{cat} \end{array}$$

We determined that this would require developing a new rule of semantic composition. Moreover, this new rule should generate as the extension of the structure above the following $\langle e,t\rangle$ expression:

(17) $[\![\ [_{\mathrm{NP}_3} \text{ gray cat }]\ ]\!] = [\lambda x : x \in D_e \,.\, x \text{ is gray and } x \text{ is a cat }]$

2.1 Introducing Predicate Modification

Considering what it required of our new rule on the basis of (16), it must be able to combine the extension of the AP and the extension of the NP_4 to derive the desired $\langle e,t\rangle$ extension of the NP_3.

(18) $[\![\text{ AP }]\!] + [\![\text{ NP}_4\ ]\!] \quad = \quad [\![\text{ NP}_3\ ]\!] \quad = \quad [\lambda x : x \in D_e \,.\, x \text{ is gray and } x \text{ is a cat }]$

Because the AP and NP_3 are of the form specified by the NN Rule, we know that the equation above can equivalently be restated as below:

(19) $[\![\text{ gray }]\!] + [\![\text{ cat }]\!] \quad = \quad [\lambda x : x \in D_e \,.\, x \text{ is gray and } x \text{ is a cat }]$

Given our notation, the value conditions of a function in our metalanguage are essentially equivalent to the extension of a predicate being applied to its argument and returning T:

(20) x is gray $\quad \approx \quad [\![\text{ gray }]\!](x) = T \quad = \quad [\lambda y : y \in D_e \,.\, y \text{ is gray }](x) = T$

(21) x is a cat $\quad \approx \quad [\![\text{ cat }]\!](x) = T \quad = \quad [\lambda y : y \in D_e \,.\, y \text{ is a cat }](x) = T$

This means that we can make the following substitutions in our metalanguage description of the extension of the NP_3.

(22) $[\![\text{ gray }]\!] + [\![\text{ cat }]\!] \quad = \quad [\lambda x : x \in D_e \,.\, [\![\text{ gray }]\!](x) = T \text{ and } [\![\text{ cat }]\!](x) = T]$

This move makes it possible to define our rules so as to take two $\langle e,t\rangle$ functions—f and g—and return the type $\langle e,t\rangle$ function that maps an entity x to T *iff* $f(x) = T$ and $g(x) = T$.

(23) $f + g \quad = \quad [\lambda x : x \in D_e \,.\, f(x) = T \text{ and } g(x) = T]$

We can formalize this idea by defining a compositional rule of **Predicate Modification**:

(24) **Predicate Modification (PM)**

If X is a node that has two daughters, Y and Z, and if $[\![\text{Y}]\!]$ and $[\![\text{Z}]\!]$ are in $D_{\langle e,t \rangle}$, then
$[\![\text{X}]\!] = [\lambda x : x \in D_e \,.\, [\![\text{Y}]\!](x) = T \text{ and } [\![\text{Z}]\!](x) = T\ ]$

This rule combines two $\langle e, t \rangle$ properties and returns the complex $\langle e, t \rangle$ property that is their conjunction. The resulting complex $\langle e, t \rangle$ expression is the function that takes an entity x as its argument and returns T *iff* each of the component functions yields T when applied to x.

Let's see how the rule of Predicate Modification computes for us our chosen extension for the NP_3.

(25) **Serial Calculation of the extension of the NP_3 *gray cat***

i.	$[\![\text{NP}_3]\!]$	=	(by PM)
ii.	$[\lambda x : x \in D_e \,.\, [\![\text{AP}]\!](x) = T \text{ and } [\![\text{NP}_4]\!](x) = T]$	=	(by NN)
iii.	$[\lambda x : x \in D_e \,.\, \underline{[\![\text{gray}]\!](x) = T} \text{ and } [\![\text{NP}_4]\!](x) = T]$	=	(by TN)
iv.	$[\lambda x : x \in D_e \,.\, \underline{[\lambda y : y \in D_e \,.\, y \text{ is gray }](x) = T} \text{ and } [\![\text{NP}_4]\!](x) = T]$	=	(by LC, def.)
v.	$[\lambda x : x \in D_e \,.\, \underline{x \text{ is gray}} \text{ and } [\![\text{NP}_4]\!](x) = T]$	=	(by NN)
vi.	$[\lambda x : x \in D_e \,.\, x \text{ is gray and } \underline{[\![\text{cat}]\!](x) = T}]$	=	(by TN)
vii.	$[\lambda x : x \in D_e \,.\, x \text{ is gray and } \underline{[\lambda y : y \in D_e \,.\, y \text{ is a cat }](x) = T}]$	=	(by LC, def.)
viii.	$[\lambda x : x \in D_e \,.\, x \text{ is gray and } \underline{x \text{ is a cat}}]$		

We will see immediately below how this extension for *gray cat* fits into the larger calculation of the truth conditions for the sentence at hand.

But we can also see look at how this calculation for the extension of *gray cat* can be computed by interpreting the inputs to Predicate Modification in parallel.

(26) **Brief Parallel Calculation of the extension of the NP_3 *gray cat***

i.	$[\![\text{NP}_3]\!]$	=	(by PM)
ii.	$[\lambda x \in D_e \,.\, [\![\text{AP}]\!](x) = T \text{ and } [\![\text{NP}_4]\!](x) = T]$	=	(by NN×2)
iii.	$[\lambda x \in D_e \,.\, \underline{[\![\text{gray}]\!](x) = T} \text{ and } \underline{[\![\text{cat}]\!](x) = T}]$	=	(by TN×2)
iv.	$[\lambda x \in D_e \,.\, \underline{[\lambda y \in D_e \,.\, y \text{ is gray }](x) = T} \text{ and } \underline{[\lambda y \in D_e \,.\, y \text{ is a cat }](x) = T}]$	=	(by LC×2, def.)
v.	$[\lambda x \in D_e \,.\, \underline{x \text{ is gray}} \text{ and } \underline{x \text{ is a cat}}\ ]$		

2.2 Predicate Modification in Semantic Proofs

The calculation of the truth conditions for *Felix is a gray cat* proceeds as follows:

(27) **Truth-conditional Statement of *Felix is a gray cat***

Felix is a gray cat is T *iff* Felix is a gray cat

(28) **Calculation of the Truth Conditions of *Felix is a gray cat***

i.	"Felix is a gray cat"	is T *iff*	(by syntax)
ii.	" $[_{S}\ [_{NP_1}\ [_{N_1}\ \textbf{Felix}]]\ [_{VP}\ [_{V}\ \textbf{is}]\ [_{NP_2}\ [_{D}\ \textbf{a}]\ [_{NP_3}\ [_{AP}\ [_{A}\ \textbf{gray}]]\ [_{NP_4}\ [_{N_4}\ \textbf{cat}]]]]]]$ "	is T *iff*	(by notation)
iii.	$[\![\text{S}]\!]$	$= T$	
iv.	*Calculation of* $[\![\text{NP}_1]\!]$		
	a. $[\![\text{NP}_1]\!]$	$=$	(by NN, TN)
	b. Felix		
v.	*Calculation of* $[\![\text{AP}]\!]$		
	a. $[\![\text{AP}]\!]$	$=$	(by NN, TN)
	b. $[\lambda x : x \in D_e \,.\, x \text{ is gray}]$		
vi.	*Calculation of* $[\![\text{NP}_4]\!]$		
	a. $[\![\text{NP}_4]\!]$	$=$	(by NN, TN)
	b. $[\lambda x : x \in D_e \,.\, x \text{ is a cat}]$		
vii.	***Calculation of*** $[\![\textbf{NP}_3]\!]$		
	a. $[\![\text{NP}_3]\!]$	$=$	(by PM)
	b. $[\lambda x : x \in D_e \,.\, [\![\text{AP}]\!](x) = T \text{ and } [\![\text{NP}_4]\!](x) = T]$	$=$	(by v.)
	c. $[\lambda x : x \in D_e \,.\, [\lambda y : y \in D_e \,.\, y \text{ is gray}](x) = T \text{ and } [\![\text{NP}_4]\!](x) = T]$	$=$	(by LC, def.)
	d. $[\lambda x : x \in D_e \,.\, x \text{ is gray and } [\![\text{NP}_4]\!](x) = T]$	$=$	(by vi.)
	e. $[\lambda x : x \in D_e \,.\, x \text{ is gray and } [\lambda y : y \in D_e \,.\, y \text{ is a cat}](x) = T]$	$=$	(by LC, def.)
	f. $[\lambda x : x \in D_e \,.\, x \text{ is gray and } x \text{ is a cat}]$		

viii. *Calculation of* $[\![\text{NP}_2]\!]$

a.	$[\![\text{NP}_2]\!]$	=	(by FA, LE, vii.)
b.	$[\![\text{a}]\!]([\![\text{NP}_3]\!])$	=	(by TN)
c.	$[\, \lambda f : f \in D_{\langle e,t \rangle} \,.\, f\,]([\![\text{NP}_3]\!])$	=	(by LC)
d.	$[\![\text{NP}_3]\!]$	=	(by vii.)
e.	$[\lambda x : x \in D_e \,.\, x$ is gray and x is a cat $]$		

ix. *Calculation of* $[\![\text{VP}]\!]$

a.	$[\![\text{VP}]\!]$	=	(by FA)
b.	$[\![\text{is}]\!]([\![\text{NP}_2]\!])$	=	(by TN)
c.	$[\, \lambda f : f \in D_{\langle e,t \rangle} \,.\, f\,]([\![\text{NP}_2]\!])$	=	(by LC)
d.	$[\![\text{NP}_2]\!]$	=	(by viii.)
e.	$[\lambda x : x \in D_e \,.\, x$ is gray and x is a cat $]$		

x.	$[\![\text{S}]\!]$	$= T$ *iff*	(by FA, ix., iv.)
xi.	$[\![\text{VP}]\!]([\![\text{NP}_1]\!])$	$= T$ *iff*	(by iv.)
xii.	$[\![\text{VP}]\!]$(Felix)	$= T$ *iff*	(by ix.)
xiii.	$[\lambda x : x \in D_e \,.\, x$ is gray and x is a cat $]$(Felix)	$= T$ *iff*	(by LC)
xiv.	Felix is gray and Felix is a cat		(nailed it)

Thus, (28i) *iff* (28xiv), or **"Felix is a gray cat" is *T iff* Felix is gray and Felix is a cat**.

By repeating the process above and in our previous lecture, we would find that our rule of Predicate Modification will derive seemingly correct truth-conditional statements for a range of **intersective** adjectives that appear in attributive position:

(29)
a. "Felix is a **furry dog**" is *T iff* Felix is **furry** and Felix is a **dog**.
b. "Felix is a **single man**" is *T iff* Felix is **single** and Felis is a **man**.
c. "Felix is a **bald hamster**" is *T iff* Felix is **bald** and Felix is a **hamster**.

3 Intersective v. Subsective Adjectives

There are, however, numerous adjectives for which Predicate Modification does not deliver an intuitively accurate set of truth conditions:

(30)
a. ?? "Richard is a **tall jockey**" is *T iff* Richard is **tall** and Richard is a **jockey**.
b. ?? "Barack is a **young ex-president**" is *T iff* Barack is **young** and Barack is a **an ex-president**.
c. ?? "Proxima Centauri is a **cold star**" is *T iff* P.C. is **cold** and P.C. is a **star**.

On the basis of this and other facts, there is good reason to suspect that the adjectives above are semantically distinct from those that we have been working with up to this point.

Moreover, their semantic differences define two natural classes of adjectives:

(31) **Intersective Adjectives**
  a. male/female
  b. American/Ethiopian
  c. blonde/brunette
  d. single/married
  e. red/yellow

(32) **Subsective Adjecives**
  a. tall/short
  b. young/old
  c. hot/cold
  d. poor/rich
  e. big/small

The idea that we will pursue in what follows is that **intersective** adjectives denote properties that hold of an entity in an "absolute" sense while **subsective** adjectives denote properties that hold of an entity in a "relative" sense when compared to a particular set of entities:

(33) **Type $\langle e,t \rangle$ Extension for Intersective Adjectives**
  $[\lambda x : x \in D_e \,.\, x \text{ is ADJ }]$

(34) **Type $\langle\langle e,t \rangle, \langle e,t \rangle\rangle$ Extension for Subsective Adjectives**
  $[\lambda f : f \in D_{\langle e,t \rangle} \,.\, [\lambda x : x \in D_e \,.\, f(x) = T \text{ and } x \text{ is ADJ } \underline{\text{for entities in } \{y : f(y) = T\}}]]$

These different semantics are responsible for several diagnostic differences between these two natural classes of predicates.

Entailment Relations. Intersective adjectives are those adjectives whose attributive usage entails their predicative usage. The attributive useage of subsective adjectives does not entail their predicate usage.

(35) **Intersective Adjectives**
  a. Dumbo is a male elephant $\Rightarrow$ Dumbo is male.
  b. Nicole is an American linguist $\Rightarrow$ Nicole is American.
  c. Gerard is a blonde waiter $\Rightarrow$ Gerard is blonde.
  d. Angie is a married politician $\Rightarrow$ Angie is married.
  e. Flounder is a yellow fish $\Rightarrow$ Flounder is yellow.

(36) **Subsective Adjectives**
  a. Chris is a short basketball player $\not\Rightarrow$ Chris is short.
  b. Barack is a young ex-president $\not\Rightarrow$ Barack is young.
  c. Proxima Centauri is a cold star $\not\Rightarrow$ Proxima Centauri is cold.
  d. Gloria is a rich professor $\not\Rightarrow$ Gloria is rich.
  e. Willy is a small whale $\not\Rightarrow$ Willy is small.

This is expected if intersective adjectives are true in some absolute sense, but subsective adjectives are true only relative to a particular set of entities denoted by a noun.

For an X. Intersective adjectives in predicate position cannot be modified by the phrase *for an X*, but this is possible for subsective adjectives.

(37) **Intersective Adjectives**

a. #Dumbo is male for an elephant.
b. #Nicole is American for a linguist.
c. #Gerard is blonde for a waiter.
d. #Angie is married for a politician.
e. #Flounder is yellow for a fish.

(38) **Subsective Adjectives**

a. Chris is short for a basketball player.
b. Barack is young for an ex-president.
c. Proxima Centauri is cold for a star.
d. Gloria is rich for a professor.
e. Willy is small for a whale.

This expected if intersective adjectives hold in an absolute sense, but subsective adjectives are true only relative to a particular set of entities denoted by a noun.

To preemptively summarize our results, adjectives come in (at least) two flavors:

(39) **Intersective Adjective**

i. uniformly predicative expressions of type $\langle e, t\rangle$
ii. properties that hold of entities in an "absolute" sense
iii. interpreted via PM: $[\lambda x_e \,.\, A(x) = T \text{ and } B(x) = T]$

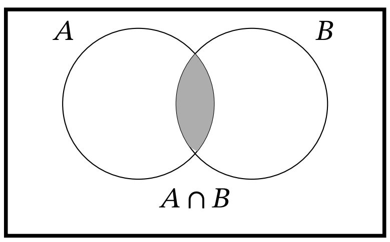


Figure 1: Illustration of the meaning of intersective adjectives.

(40) **Subsective Adjective**

i. uniformly modificational expressions of type $\langle\langle e, t\rangle, \langle e, t\rangle\rangle$
ii. properties that hold of entities in an "relative" sense
iii. interpreted via FA: $[\lambda A_{\langle e,t\rangle} \,.\, [\lambda x_e \,.\, A(x) = T \text{ and } x \text{ is } B \text{ for entities in } \{y : A(y) = T\}]]$

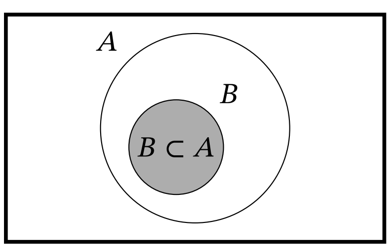


Figure 2: Illustration of the meaning of subsective adjectives.

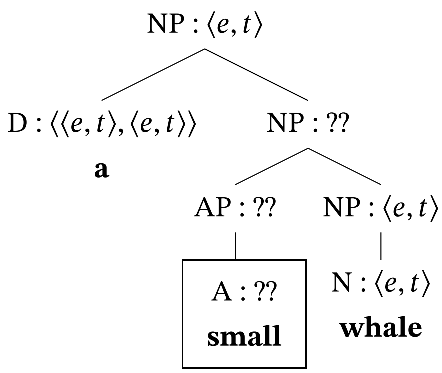
4 The Semantics of Subsective Adjectives

Taking stock, we have determined that there is a class of subsective adjectives for which a type $\langle e,t\rangle$ denotation and composition via Predicate Modification do not deliver an intuitively correct meaning.

This leaves us with the questions of how we can and should incorporate subsective adjectives into our semantic system.

(41) a. Willy is **a small whale**.

b.



How to Move Forward. In order to provide some answers, we will need to do a few things:

- We will need to determine what semantic type subsective adjectives must have.
- We will need to formulate an accurate set of truth conditions for subsective adjectives and an extension that delivers those truth conditions.
- We will need to demonstrate that our semantic system can compute the correct truth conditions of sentences containing a subsective adjective.

4.1 The Semantic Type of Subsective Adjectives

1. Known Semantic Types. We start by listing out the semantic types we know and annotating the syntactic representation with this information.

(42) **Known Semantic Types in (43)**

a. $[\![\text{S}]\!] \in D_t$

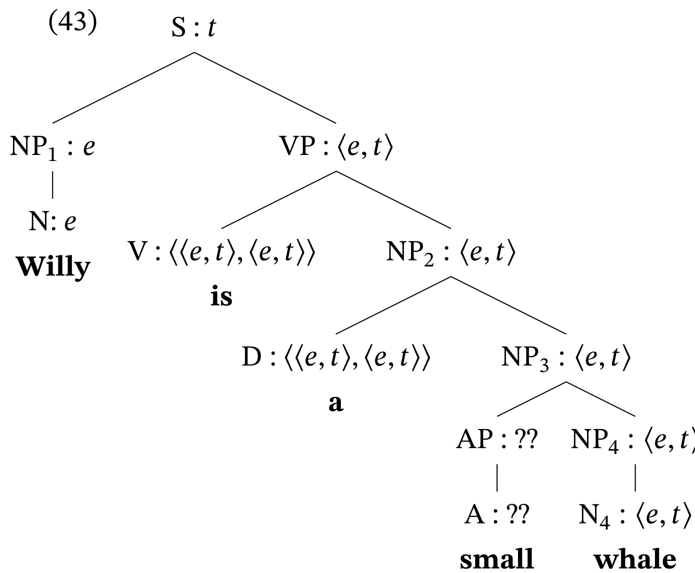
b. $[\![\text{Willy}]\!] \in D_e$

c. $[\![\text{is}]\!] \in D_{\langle\langle e,t\rangle,\langle e,t\rangle\rangle}$

b. $[\![\text{a}]\!] \in D_{\langle\langle e,t\rangle,\langle e,t\rangle\rangle}$

b. $[\![\text{whale}]\!] \in D_{\langle e,t\rangle}$

(43)



2. Reasoning out Other Types.

- Because the NP_1 and NP_4 nodes fit the description of the NN Rule, they have the same semantic type as their daughters.
- Because the $[\![\, \mathrm{S} \,]\!]$ is type t and the $[\![\, \mathrm{NP}_1 \,]\!]$ is type e, FA entails that the $[\![\, \mathrm{VP} \,]\!]$ is of type $\langle e, t\rangle$.
- Because the $[\![\, \mathrm{VP} \,]\!]$ is type $\langle e, t\rangle$ and $[\![\, \mathrm{is} \,]\!]$ is type $\langle\langle e, t\rangle, \langle e, t\rangle\rangle$, FA entails that the $[\![\, \mathrm{NP}_2 \,]\!]$ is of type $\langle e, t\rangle$.
- Because the $[\![\, \mathrm{NP}_2 \,]\!]$ is type $\langle e, t\rangle$ and $[\![\, \mathrm{a} \,]\!]$ is type $\langle\langle e, t\rangle, \langle e, t\rangle\rangle$, FA entails that the $[\![\, \mathrm{NP}_3 \,]\!]$ is of type $\langle e, t\rangle$.

3. Reasoning out the Semantic Type of the A *small*. We appeal to the known semantic types, as well as the rules of composition, to determine that the A must be of type $\langle\langle e, t\rangle, \langle e, t\rangle\rangle$.

- The NP_3 node fits the structural description specified by Functional Application and Predicate Modification. So, $[\![\, \mathrm{AP} \,]\!]$ must combine with $[\![\, \mathrm{NP}_4 \,]\!]$ via one of these rules.

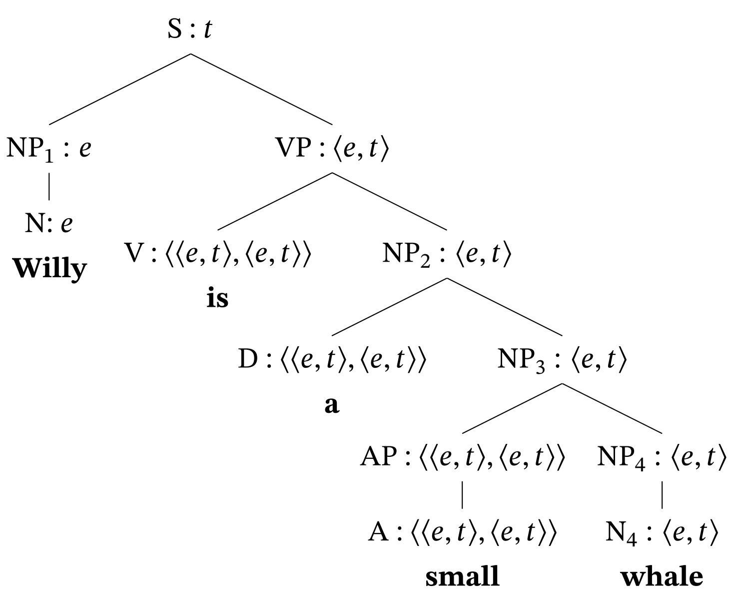
(44) $[\![\, \mathrm{AP} \,]\!] + [\![\, \mathrm{NP}_4 \,]\!] = [\![\, \mathrm{NP}_3 \,]\!]$

- We determined in section 3 that Predicate Modification does not deliver the correct results for subsective adjectives. So, $[\![\, \mathrm{AP} \,]\!]$ and $[\![\, \mathrm{NP}_4 \,]\!]$ must be composed via Functional Application.
- Because $[\![\, \mathrm{NP}_3 \,]\!]$ is type $\langle e, t\rangle$ and $[\![\, \mathrm{NP}_4 \,]\!]$ is type $\langle e, t\rangle$, $[\![\, \mathrm{AP} \,]\!]$ must be a function of type $\langle\langle e, t\rangle, \langle e, t\rangle\rangle$ that takes $[\![\, \mathrm{NP}_4 \,]\!]$ as its argument and returns the type $\langle e, t\rangle$ $[\![\, \mathrm{NP}_3 \,]\!]$.

(45) $[\![\, \mathrm{AP} \,]\!]([\![\, \mathrm{NP}_4 \,]\!]) = [\![\, \mathrm{NP}_3 \,]\!]$

- Because $[\![\, \mathrm{AP} \,]\!]$ is equivalent to $[\![\, \mathrm{A} \,]\!]$, the extension of the A *small* must be an expression of type $\langle\langle e, t\rangle, \langle e, t\rangle\rangle$.

(46)



4.2 The Extension of Subsective Adjectives

We now know what kind of expression the A *small* must be. It is a type $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$ function, meaning it maps functions from entities to truth values to functions from entities to truth values.

We therefore need to develop a lexical entry for *small* that does the following:

- assigns as its extension a function of type $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$ and
- allows our system to derive the appropriate truth-conditional statement for its meaning.

Regarding the appropriate truth conditions, we can take seriously the observation made above that subsective adjectives hold of some entity in a "relative sense".

More specifically, these adjectives hold of some entity relative to a specific set of entities, and it's always the N they modify that provides this set.

(47) a. "tall **jockey**" = "tall <u>for a **jockey**</u>"
b. "cold **star**" = "cold <u>for a **star**</u>"
c. "small **whale**" = "small <u>for a **whale**</u>"

With this, we can provide significantly more accurate truth-conditional statements like those below:

(48) a. "Richard is a tall jockey" is T *iff*
Richard is a jockey and Richard is tall for entities in $\{y : y \text{ is a jockey}\}$.
b. "Proxima Centauri is a cold star" is T *iff*
P.C. is a star and P.C. is cold for entities in $\{y : y \text{ is a star}\}$.
c. "Willy is a small whale" is T *iff*
Willy is a whale and Willy is small for entities in $\{y : y \text{ is a whale}\}$.

Now, the progress that we are making with our truth conditions dovetails nicely with our conclusion from section 4.1 that subsective adjectives must take a type $\langle e,t\rangle$ expression as an argument and return a type $\langle e,t\rangle$ expression as its value.

The NP that a subsective adjective modifies, as a type $\langle e,t\rangle$ expression, can provide this argument. This function can be applied some entity and determine the set of entities for comparison.

The type $\langle e,t\rangle$ expression returned as the value of the extension of the subsective adjective, like any other predicate, can combine with the subject to produce a truth value.

Using some equivalencies established previously, we can accomplish all of this with lexical entries for subsective adjectives that like those shown below:

(49) **Sample lexical entries for subsective adjectives**
a. $[\![\text{ tall }]\!] = [\lambda f \in D_{\langle e,t\rangle} . [\lambda x \in D_e . f(x) = T \text{ and } x \text{ is tall for entities in } \{y : f(y) = T\}\,]]$
b. $[\![\text{ cold }]\!] = [\lambda f \in D_{\langle e,t\rangle} . [\lambda x \in D_e . f(x) = T \text{ and } x \text{ is cold for entities in } \{y : f(y) = T\}\,]]$
c. $[\![\text{ small }]\!] = [\lambda f \in D_{\langle e,t\rangle} . [\lambda x \in D_e . f(x) = T \text{ and } x \text{ is small for entities in } \{y : f(y) = T\}\,]]$

4.3 A Semantic Proof

Let's check that our semantic system is able to correctly compute the meaning of sentences with subsective adjectives. The lexical entries we have for the component parts of the relevant sentence are as follows:

(50) **Lexical entries**

a. $[\![\text{Willy}]\!] = \text{Willy}$

b. $[\![\text{is}]\!] = [\, \lambda f \in D_{\langle e,t\rangle} \,.\, f \,]$

c. $[\![\text{a}]\!] = [\, \lambda f \in D_{\langle e,t\rangle} \,.\, f \,]$

d. $[\![\text{small}]\!] = [\lambda f \in D_{\langle e,t\rangle} \,.\, [\lambda x \in D_e \,.\, f(x) = T \text{ and } x \text{ is small for entities in } \{y : f(y) = T\}\,]]$

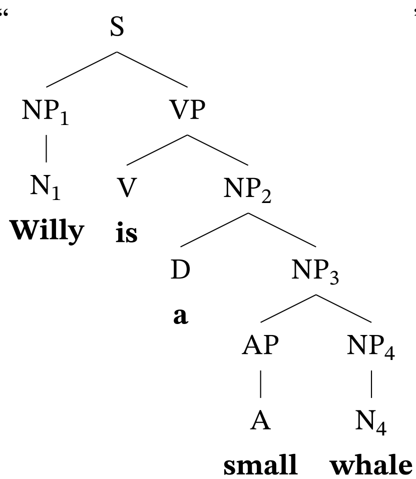
e. $[\![\text{whale}]\!] = [\lambda x \in D_e \,.\, x \text{ is a whale}\,]$

The truth-conditional statement we wish to derive is:

(51) **Truth-conditional Statement of *Willy is a small whale***
Willy is a small whale is T *iff* Willy is a whale and Willy is small for entities in $\{y : y \text{ is a whale}\}$

(52) **Calculation of the Truth Conditions of *Willy is a small whale***

i. "Willy is a small whale" is T *iff* (by syntax)

ii. " 

" is T *iff* (by notation)

iii. $[\![\text{S}]\!]$ $= T$

iv. *Calculation of* $[\![\text{NP}_1]\!]$

a. $[\![\text{NP}_1]\!]$ $=$ (by NN, TN)

b. Willy

v. *Calculation of* $[\![\text{AP}]\!]$

a. $[\![\text{AP}]\!]$ $=$ (by NN, TN)

b. $[\lambda f \in D_{\langle e,t\rangle} \,.\, [\lambda x \in D_e \,.\, f(x) = T$ and
x is small for entities in $\{y : f(y) = T\}\,]]$

vi. *Calculation of* $[\![$ NP$_4$ $]\!]$

a. $[\![\text{NP}_4]\!]$ $=$ (by NN, TN)

b. $[\lambda x : x \in D_e \, . \, x \text{ is a whale }]$

vii. ***Calculation of*** $[\![$ **NP$_3$** $]\!]$

a. $[\![\text{NP}_3]\!]$ $=$ (by FA, v., vi.)

b. $[\![\text{AP}]\!]([\![\text{NP}_4]\!])$ $=$ (by v.)

c. $[\lambda f \in D_{\langle e,t \rangle} \, . \, [\lambda x \in D_e \, . \, f(x) = T \text{ and } x \text{ is small for entities in } \{y : f(y) = T\} \,]]([\![\text{NP}_4]\!])$ $=$ (by LC)

d. $[\lambda x \in D_e \, . \, [\![\text{NP}_4]\!](x) = T \text{ and } x \text{ is small for entities in } \{y : [\![\text{NP}_4]\!](y) = T\} \,]$ $=$ (by vi.)

e. $[\lambda x \in D_e \, . \, [\lambda z \in D_e \, . \, z \text{ is a whale }](x) = T \text{ and } x \text{ is small for entities in } \{y : [\lambda z \in D_e \, . \, z \text{ is a whale }](y) = T\} \,]$ $=$ (by LC, def.)

f. $[\lambda x \in D_e \, . \, x \text{ is a whale and } x \text{ is small for entities in } \{y : y \text{ is a whale }\} \,]$

viii. *Calculation of* $[\![$ NP$_2$ $]\!]$

a. $[\![\text{NP}_2]\!]$ $=$ (by FA, LE, vii.)

b. $[\![\text{a}]\!]([\![\text{NP}_3]\!])$ $=$ (by TN)

c. $[\, \lambda f \in D_{\langle e,t \rangle} \, . \, f \,]([\![\text{NP}_3]\!])$ $=$ (by LC)

d. $[\![\text{NP}_3]\!]$ $=$ (by vii.)

e. $[\lambda x \in D_e \, . \, x \text{ is a whale and } x \text{ is small for entities in } \{y : y \text{ is a whale }\} \,]$

ix. *Calculation of* $[\![$ VP $]\!]$

a. $[\![\text{VP}]\!]$ $=$ (by FA)

b. $[\![\text{is}]\!]([\![\text{NP}_2]\!])$ $=$ (by TN)

c. $[\, \lambda f \in D_{\langle e,t \rangle} \, . \, f \,]([\![\text{NP}_2]\!])$ $=$ (by LC)

d. $[\![\text{NP}_2]\!]$ $=$ (by viii.)

e. $[\lambda x \in D_e \, . \, x \text{ is a whale and } x \text{ is small for entities in } \{y : y \text{ is a whale }\} \,]$

x. $[\![\text{S}]\!]$ $= T$ *iff* (by FA, ix., iv.)

xi. $[\![\text{VP}]\!]([\![\text{NP}_1]\!])$ $= T$ *iff* (by iv.)

xii. $[\![\text{VP}]\!](\text{Willy})$ $= T$ *iff* (by ix.)

xiii. $[\lambda x \in D_e \, . \, x \text{ is a whale and } x \text{ is small for entities in } \{y : y \text{ is a whale }\} \,](\text{Willy})$ $= T$ *iff* (by LC)

xiv. Willy is a whale and Willy is small for entities in $\{y : y \text{ is a whale }\}$ (nailed it)

Thus, (50i) *iff* (50xiv), or **"Willy is a small whale" is T *iff* Willy is a whale and Willy is small for entities in** $\{\boldsymbol{y} : \boldsymbol{y}$ **is a whale** $\}$.

By repeating the process above and in our previous lecture, we would find that our lexical entries for subsective adjectives produce the correct truth-conditional statements for a range of **subsective** adjectives that appear in attributive position:

(53) a. "Richard is a tall jockey" is T *iff*
Richard is a jockey and Richard is tall for entities in $\{y : y$ is a jockey$\}$.

b. "Proxima Centauri is a cold star" is T *iff*
P.C. is a star and P.C. is cold for entities in $\{y : y$ is a star$\}$.

c. "Barack is a young ex-president" is T *iff*
Barack is an ex-president and Barack is young for an entity in $\{y : y$ is an ex-president$\}$.

Importantly, these truth conditions avoid the undesirable result that we initially observed with an analysis that relied on Predicate Modification. For example, just because Richard it a tall jockey it's not entailed that he is tall in an absolute sense.

(54) a. ?? "Richard is a tall jockey" is T *iff* Richard is tall and Richard is a jockey.

b. ?? "Proxima Centauri is a cold star" is T *iff* P.C. is cold and P.C. is a star.

c. ?? "Barack is a young ex-president" is T *iff* Barack is young and Barack is a an ex-president.

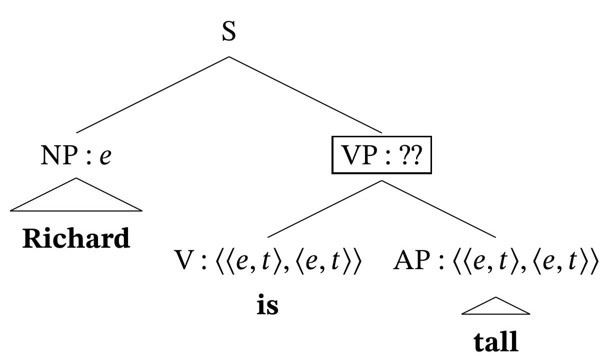
5 Subsective Adjectival Predicates

In what has preceded we have uncovered a class of subsective adjectives, which are type $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$ expressions that take the NP they syntactically modify as a semantic argument.

But, if this is the case, it's not clear that we should expect these adjectives to appear in predicative position. These structures should produce a type mismatch and be uninterpretable, contrary to fact.

(55) a. Richard is tall.

b.



We don't have a rule of composition that is able to compute the meaning of an expression formed by composing two type $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$ expressions.

One possible solution to this apparent problem for our system would be to assume that at least some adjectives are systematically ambiguous between an intersective and subsective interpretation.

More specifically, an adjective like *tall* can have either of the interpretations below:

(56) $[\![\text{tall}_{int}]\!] = [\lambda x \in D_e \,.\, x \text{ is tall}]$

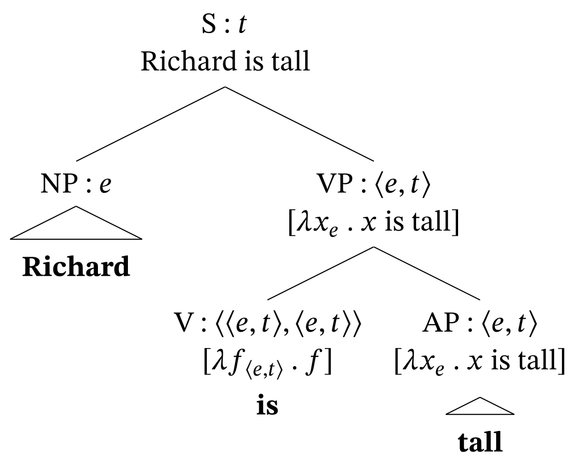
(57) $[\![\text{tall}_{sub}]\!] = [\lambda f \in D_{\langle e,t \rangle} \,.\, [\lambda x \in D_e \,.\, f(x) = T \text{ and } x \text{ is tall for entities in } \{y : f(y) = T\}]]$

It is possible to identify at least two prediction from this analysis.

Intersective Predicates. Because our rules of composition can only interpret type $\langle e,t \rangle$ intersective adjectives in predicate position, adjectival predicates should only permit an “absolute” reading.

(58) a. Richard is tall.

b.



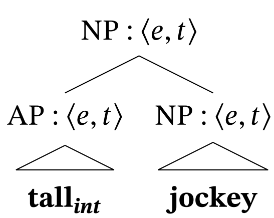
Thus, (58) should be compatible with the truth conditions in (59). But it’s not entirely clear that the relative reading is entirely unavailable, as (60) hints at.

(59) “Richard is tall” is T *iff* Richard is tall (in an absolute sense).

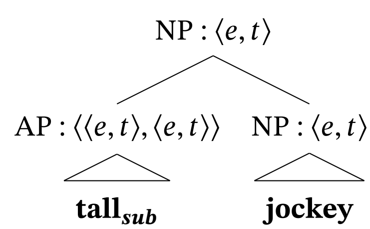
(60) “Willy is small” is T *iff* Willy is small for a whale.

Ambiguous Modifiers. Because our rules of composition can interpret attributive adjectives of type $\langle e,t \rangle$ via Predicate Modification and attributive adjectives of type $\langle \langle e,t \rangle, \langle e,t \rangle \rangle$ via Functional application, attributive adjectives should permit either an “absolute” or a “relative” reading.

(61) **Intersective = “Absolute”**



(62) **Subsective = “Relative”**



When a type $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$ subsective adjective composes with a noun, as in (63), the result is expected to be a "relative" interpretation for that adjective, as we have seen in section 4.

(63) a. Richard is a tall$_{sub}$ jockey.

b.

S : t
Richard is a jockey and Richard is tall for entities in $\{y : y$ is a jockey$\}]]$

NP$_1$: e
Richard

VP : $\langle e,t\rangle$
$[\lambda x_e$. x is a jockey and x is tall for entities in $\{y : y$ is a jockey$\}]]$

V : $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$
$[\lambda f_{\langle e,t\rangle} . f]$
is

NP$_2$: $\langle e,t\rangle$
$[\lambda x_e$. x is a jockey and x is tall for entities in $\{y : y$ is a jockey$\}]]$

D : $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$
$[\lambda f_{\langle e,t\rangle} . f]$
a

NP$_3$: $\langle e,t\rangle$
$[\lambda x_e$. x is a jockey and x is tall for entities in $\{y : y$ is a jockey$\}]]$

AP : $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$
$[\lambda f_{\langle e,t\rangle} . [\lambda x_e . f(x) = T$ and x is tall for entities in $\{y : f(y) = T\}]]$
tall$_{sub}$

NP$_4$: $\langle e,t\rangle$
$[\lambda x_e$. x is a jockey$]$
jockey

Thus, (63) is compatible with the truth condition in (64):

(64) "Richard is a tall jockey" is T *iff*
Richard is a jockey and Richard is tall for entities in $\{y : y$ is a jockey$\}$

When a type $\langle e,t \rangle$ intersective adjective composes with a noun, as in (65), the result should be an "absolute" interpretation for that adjective.

(65) a. Richard is a tall jockey.

b.

- S : t
 Richard is tall and Richard is a jockey
  - $NP_1 : e$
    - **Richard**
  - VP : $\langle e,t \rangle$
 $[\lambda x_e \,.\, x$ is a tall and x is a jockey$]$
    - V : $\langle \langle e,t \rangle, \langle e,t \rangle \rangle$
 $[\lambda f_{\langle e,t \rangle} \,.\, f]$
 is
    - $NP_2 : \langle e,t \rangle$
 $[\lambda x_e \,.\, x$ is a tall and x is a jockey$]$
      - D : $\langle \langle e,t \rangle, \langle e,t \rangle \rangle$
 $[\lambda f_{\langle e,t \rangle} \,.\, f]$
 a
      - $NP_3 : \langle e,t \rangle$
 $[\lambda x_e \,.\, x$ is a tall and x is a jockey$]$
        - AP : $\langle e,t \rangle$
 $[\lambda x_e \,.\, x$ is tall$]$
          - $\mathbf{tall}_{\boldsymbol{int}}$
        - $NP_4 : \langle e,t \rangle$
 $[\lambda x_e \,.\, x$ is a jockey$]$
          - **jockey**

Thus, (65) should be compatible with the truth condition in (66):

(66) "Richard is a tall jockey" is *T iff* Richard is tall and Richard is a jockey

To the extent that it can be demonstrated that both interpretations are available, we would have evidence for the type of analysis based on ambiguity that is being presented here.

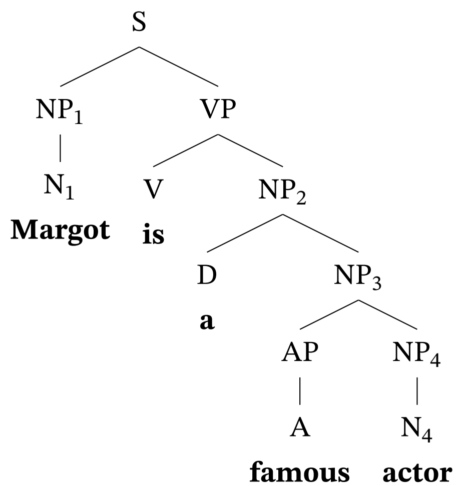
The following type of sentence may serve to demonstrate that both interpretations are available. If (67) has an interpretation that is not simply contradictory and incoherent, then an absolute meaning must be available for *tall* in predicate position.

(67) Richard isn't a tall jockey. He's just tall for a jockey.

6 Practice

Exercise. Please provide a semantic proof of the truth-conditions generated from the represetnation below:

(68)



In order to do this, we will need to provide the extensions of the lexical items and provide a step-by-step proof of the intuitive truth conditions.