Adjectival Modification

Lecture 10

February 13, 2024

Announcements:

This lecture is supplemented by the following readings:

• Heim & Kratzer: Ch.4 (63-67)

Your fourth homework assignment is available and is due on February 20th.

1 Introduction

We currently have a semantic system for interpreting sentences that relies on the following set of rules:

(1) Functional Application (FA)

If X is a node that has two daughters, Y and Z, and if [Y] is a function whose domain contains [Z], then [X] = [Y] ([Z]).

(2) Non-Branching Nodes (NN) Rule

If X is a non-branching node that has Y as its daughter, then $[\![X]\!] = [\![Y]\!]$

(3) Terminal Nodes (TN) Rule

If X is a terminal node, then [X] is specified in the lexicon.

These rules are able to interpret structures built from lexical entries like the following:

(4) **Proper Nouns as entities**

 $[\![NAME]\!]$ = the thing referred to with NAME

(5) Type $\langle e, t \rangle$ expressions

- a. $[VERB_{intrans}] = [\lambda x : x \in D_e . T iff x VERBs]$
- b. $[ADJECTIVE] = [\lambda x : x \in D_e . T iff x is ADJECTIVE]$
- c. $\| \text{NOUN} \| = [\lambda x : x \in D_e . T \text{ iff } x \text{ is a NOUN }]$

(6) Type $\langle e, \langle e, t \rangle \rangle$ expressions

$$[\![VERB_{trans}]\!] = [\lambda x : x \in D_e . [\lambda y : y \in D_e . T iff y VERBs x]]$$

(7) Type $\langle \langle e, t \rangle, \langle e, t \rangle \rangle$ expressions

a.
$$[\![is]\!] = [\lambda f : f \in D_{\langle e,t \rangle} . f]$$

b.
$$[a] = [\lambda f : f \in D_{\langle e, t \rangle} \cdot f]$$

We have recently turned our attention to the semantics of adjectives, adjectival modification, and more complex nominal expressions. During our last few lectures we focused on interpreting predicate adjectives and predicate nominals:

(8) Felix is **gray**.

(9) Felix is a cat.

Today we will work on developing our semantic system so that it is able to compute the meaning of sentences with **attributive adjectives**. This is the term used to refer to adjectives that appear inside of nominal expressions to modify a noun, like in (10):

(10) Felix is a **gray** cat.

We will find that, as things currently are, our system is unable to interpret such sentences. However, given our previous results, we will see that it is possible to introduce the semantic rule of **Predicate Modification** to serve our purposes.

(11) **Predicate Modification (PM)**

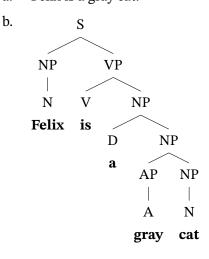
If X is a node that has two daughters, Y and Z, and if
$$[\![Y]\!]$$
 and $[\![Z]\!]$ are in $D_{\langle e,t\rangle}$, then $[\![X]\!] = [\lambda x : x \in D_e$. T iff $[\![Y]\!](x) = T$ and $[\![Z]\!](x) = T$

This rules effectively combines two type $\langle e, t \rangle$ expressions into a complex type $\langle e, t \rangle$ expression.

2 A Semantic Type "Mismatch"

Toward the goal of providing a semantic system that can compute the meaning of the sentences at hand, let us first make explicit out syntactic assumptions.

(12) a. Felix is a gray cat.

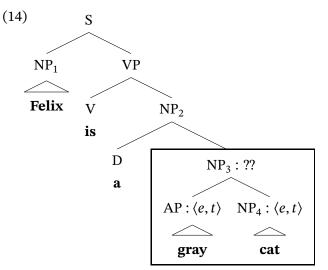


By considering the semantic composition of this expression, we find that the set of semantic rules that we have to interpret it are insufficient for doing so.

More specifically, we can observe in (14) that we are not able to compute the meaning of the node NP₃.

(13) Known Semantic Types in (12)

- a. [S] $\in D_t$
- b. $[\![\text{Felix}]\!] \in D_e$
- c. [is] $\in D_{\langle\langle e,t\rangle,\langle e,t\rangle\rangle}$
- b. [a] $\in D_{\langle\langle e,t\rangle,\langle e,t\rangle\rangle}$
- b. $[\![gray]\!] \in D_{\langle e,t \rangle}$
- b. $[\![\operatorname{cat}]\!] \in D_{\langle e,t \rangle}$



A Semantic Type "Mismatch" at NP₃. By considering the known semantic types and our rules of compositions, we can see that, at present, the extensions of the AP and NP₄ cannot be composed.

- The NP₃ node fits the structural description specified by FA.
- But the daughters of NP₃—the AP and the NP₄—both denote $\langle e, t \rangle$ functions. So, neither can take the other as an argument.
- Consequently, FA cannot be applied to interpret NP₃.
- Thus, we are not able to compute $[\![NP_3]\!]$.

How to Move Forward. In order to solve this problem, we will need to do a few things:

- We will need to determine an accurate set of truth conditions for the sentence in (12) that we want our system to compute.
- We will need to determine what the extension of NP₃ has to be (i.e., how it contributes to the truth conditions of the utterance.
- We will need to propose a new rule of semantic composition to derive the extension of the NP₃ from its components parts: the AP *gray* and the NP₄ *cat*.

3 The Semantics of Attributive Adjectives

Toward determining the truth-conditional contributions of the NP₃ gray cat, we might first consider the truth conditions that we previously assigned to its component parts: the adjective and the noun.

- (15) "Felix is gray" is T iff Felix is gray.
- (16) "Felix is a **cat**" is T iff Felix is a **cat**.

One way of thinking about our sentence at hand is that *a gray cat*, as a predicate, is applying both of these properties *gray* and *cat* to the subject NP *Felix*.

Taking this observation seriously, the statement in (17) provides an intuitive and seemingly correct set of truth conditions for the sentence.

(17) "Felix is a **gray cat**" is *T iff* Felix is **gray** and Felix is a **cat**.

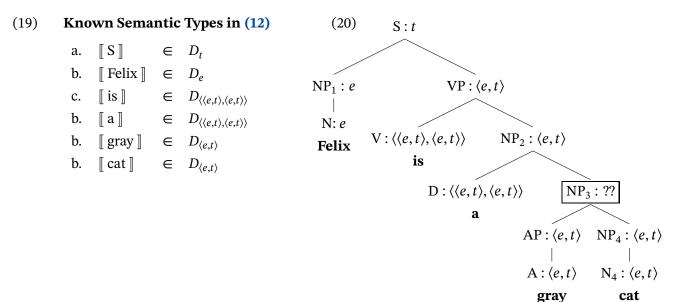
These truth conditions also seem to put us on the right track for a range of attributive adjectives.

- (18) a. "Felix is a **furry cat**" is *T iff* Felix is **furry** and Felix is **a cat**.
 - b. "Felix is a **male cat**" is *T iff* Felix is **male** and Felix is **a cat**.
 - c. "Felix is a **single cat**" is *T iff* Felix is **single** and Felix is **a cat**.

With these truth conditions in hand, we can start working toward the extension of the NP_3 . This requires that we know the semantic type that the NP_3 must have in order to ensure composition.

3.1 The Semantic Type of the NP₃ gray cat

1. Known Semantic Types. We start by listing out the semantic types we know and annotating the syntactic representation with this information.



2. Reasoning out Other Types.

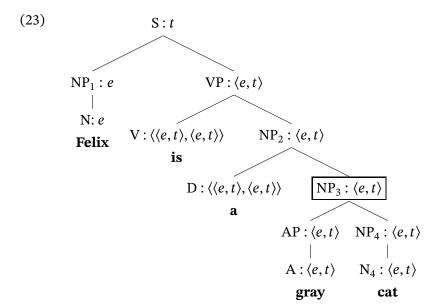
- Because the NP₁, AP, and NP₄ nodes fit the description of the NN Rule, they have the same semantic type as their daughters.
- Because the $[\![S]\!]$ is type t and the $[\![NP_1]\!]$ is type e, FA entails that the $[\![VP]\!]$ is of type $\langle e, t \rangle$.
- Because the $[\![VP]\!]$ is type $\langle e,t\rangle$ and $[\![is]\!]$ is type $\langle \langle e,t\rangle, \langle e,t\rangle\rangle$, FA entails that the $[\![NP_2]\!]$ is of type $\langle e,t\rangle$.

- **3. Reasoning out the Semantic Type of the NP₃.** We appeal to the known semantic types, as well as the rules of composition, to determine that the NP₃ must be of type $\langle e, t \rangle$.
 - The NP₂ node fits the structural description for Functional Application. So, the extension of the determiner must combine with the extension of the NP₃ to return the extension of the NP₂.

$$[21) [a] + [NP3] = [NP2]$$

• Because the determiner is a function of type $\langle \langle e, t \rangle, \langle e, t \rangle \rangle$, the extension of the determiner must take the NP₃ as its argument to return the type $\langle e, t \rangle$ extension of the NP₂ as its value.

• Thus, the NP₃ must be an expression of type $\langle e, t \rangle$.



3.2 The Extension of the NP₃ gray cat

We now know what kind of expression the NP₃ gray cat must be. It is a type $\langle e, t \rangle$ function, meaning it maps entities to truth values.

We therefore need to develop a lexical entry for gray cat that does the following:

- assigns as its extension a function of type $\langle e, t \rangle$ and
- allows our system to derive the following truth-conditional statement:
 - (24) "Felix is a gray cat" is *T iff* Felix is gray and Felix is a cat.

Some Preliminary Reasoning about the Extension of NP $_3$. On the basis of what we know, we can appreciate that the extension of the VP will be equivalent to the extension of the NP $_3$.

• We can determine that the type $\langle e, t \rangle$ [VP] will be equivalent to the type $\langle e, t \rangle$ [NP₂] on account of the copula's extension as an identity function.

- We can determine that the type $\langle e, t \rangle$ [NP₂] will be equivalent to the type $\langle e, t \rangle$ [NP₃] on account of the determiner's extension as an identity function.
- From these two points it follows that $[VP] = [NP_3]$.
 - (25) The extension of the VP is identical to the extension of the NP₃

Reasoning out the Extension of the NP₃. Given the results above, determining the meaning of the type $\langle e, t \rangle$ VP *is a gray cat* will deliver the meaning of the NP₃ *gray cat*.

- Considering sentences like those in (26), the key generalization is that the VP *is a gray cat* systematically combines with entities to generate sentences with the meaning in (27).
 - (26) a. "Felix is a gray cat" is *T iff* Felix is gray and Felix is a cat.
 - b. "Mittens is a gray cat" is *T iff* Mittens is gray and Mittens is a cat.
 - c. "Luna is a gray cat" is *T iff* Luna is gray and Luna is a cat.
 - (27) [[S] NAME is a gray cat] = T iff NAME is gray and NAME is a cat
- We know from section 3.1 that [[VP] is a gray cat] [VP] is of type $\langle e, t \rangle$. Consequently, our rule of FA entails that the following equivalency holds:
- If follows from the previous two points that:
 - (29) $[V_P]$ is a gray cat $[V_P]$ is a gray cat $[V_P]$ is a gray and NAME is a cat
- Therefore, [[VP is a gray cat]] is a type ⟨e, t⟩ function that takes an argument x and yields T iff x is gray and x is a cat.
 - (30) $[[V_P \text{ is a gray cat }]] = [\lambda x : x \in D_e . T \text{ iff } x \text{ is a gray and } x \text{ is a cat }]$
- Because $[\![[v_P \text{ is a cat }]]\!] = [\![[v_{P_3} \text{ gray cat }]]\!]$, then:
 - (31) $[[NP_3 \text{ gray cat }]] = [\lambda x : x \in D_e . T \text{ iff } x \text{ is a gray and } x \text{ is a cat }]$

4 The Rule of Predicate Modification

Our current goal is to develop our semantic system so that it can interpret sentences that contain attributive adjectives, like in (32).

(32) Felix is a **gray** cat.

By building upon our previous results for adjectives and common nouns, we have up to this point motivated the following extension for the NP *gray cat*:

(33)
$$[[NP_3 \text{ gray cat }]] = [\lambda x : x \in D_e . T \text{ iff } x \text{ is gray and } x \text{ is a cat }]$$

What we will still require is a new compositional rule that is capable of deriving the extension of the NP_3 node in a structure like (34). As we saw above, our current set of semantic rules are capable of this.

(34)
$$\begin{array}{c|ccc}
 & NP_3 : \langle e, t \rangle \\
 & AP : \langle e, t \rangle & NP_4 : \langle e, t \rangle \\
 & & | & | \\
 & A : \langle e, t \rangle & N_4 : \langle e, t \rangle \\
 & & \mathbf{gray} & \mathbf{cat}
\end{array}$$

Considering what it required of our new rule on the basis of (34), it must be able to combine the extension of the AP and the extension of the NP₄ to derive the desired $\langle e, t \rangle$ extension of the NP₃.

(35)
$$[AP] + [NP_4] = [NP_3] = [\lambda x : x \in D_e . T iff x is gray and x is a cat]$$

Because the AP and NP₃ are of the form specified by the NN Rule, we know that the equation above can equivalently be restated as below:

(36)
$$\| \operatorname{gray} \| + \| \operatorname{cat} \| = [\lambda x : x \in D_e . T \text{ iff } x \text{ is gray and } x \text{ is a cat }]$$

Now, given our notation, the value conditions of a function in our metalanguage are essentially equivalent to the extension of a predicate being applied to its argument and returning *T*:

(37)
$$x \text{ is gray } \approx \| \operatorname{gray} \|(x) = T = [\lambda y : y \in D_e . T \text{ iff } y \text{ is gray }](x) = T$$

(38)
$$x \text{ is a cat } \approx \| \cot \|(x) = T \| = [\lambda y : y \in D_e] \cdot T \text{ iff } y \text{ is a cat }](x) = T$$

This means that we can make the following substitutions in our metalanguage description of the extension of the NP₃.

(39)
$$\| \operatorname{gray} \| + \| \operatorname{cat} \| = [\lambda x : x \in D_e . T \text{ iff } \| \operatorname{gray} \| (x) = T \text{ and } \| \operatorname{cat} \| (x) = T]$$

This move makes it possible to have our new rule take two $\langle e, t \rangle$ functions—f and g—and return the type $\langle e, t \rangle$ function that maps an entity x to T iff f(x) = T and g(x) = T.

We can formalize this idea by defining a compositional rule of **Predicate Modification**:

(40) **Predicate Modification (PM)**

If X is a node that has two daughters, Y and Z, and if
$$[\![Y]\!]$$
 and $[\![Z]\!]$ are in $D_{\langle e,t\rangle}$, then $[\![X]\!] = [\lambda x : x \in D_e$. $[\![Y]\!](x) = T$ and $[\![Z]\!](x) = T$

This rule combines two $\langle e, t \rangle$ properties and returns the complex $\langle e, t \rangle$ property that is their conjunction. The resulting complex $\langle e, t \rangle$ expression is the function that takes an entity x as its argument and returns T iff each of the component functions yields T when applied to x.

Let's see how the rule of Predicate Modification computes for us our chosen extension for the NP₃.

(41) Calculation of the extension of the NP₃ gray cat

i.
$$[NP_3]$$
 (by PM)

ii.
$$[\lambda x : x \in D_e . T iff [AP](x) = T \text{ and } [NP_4](x) = T]$$
 (by NN)

iii.
$$[\lambda x : x \in D_e . T \text{ iff } [\![\text{gray }]\!](x) = T \text{ and } [\![\text{NP}_4]\!](x) = T]$$
 (by TN)

iv.
$$[\lambda x : x \in D_e . T \text{ iff } [\lambda y : y \in D_e . T \text{ iff } y \text{ is gray }](x) = T \text{ and } [NP_4](x) = T]$$
 (by LC, def.)

v.
$$[\lambda x : x \in D_e . T \text{ iff } x \text{ is gray and } [NP_4](x) = T]$$
 (by NN)

vi.
$$[\lambda x : x \in D_e . T \text{ iff } x \text{ is gray and } [\![\text{cat }]\!](x) = T]$$
 (by TN)

vii.
$$[\lambda x : x \in D_e . T \text{ iff } x \text{ is gray and } [\lambda y : y \in D_e . T \text{ iff } y \text{ is a cat }](x) = T]$$
 (by LC, def.)

viii. $[\lambda x : x \in D_e . T \text{ iff } x \text{ is gray and } \underline{x \text{ is a cat}}]$

We will see in the following section how this extension for *gray cat* fits into the larger calculation of the truth conditions for the sentence at hand.

But first, now is a perfectly good time to demonstrate the utility of our abbreviated lambda notation for functions. While the notations below are equivalent, (43) sacrifices several recoverable redundancies in the name of brevity and legibility.

$$[NP_3] = [\lambda x : x \in D_e . T iff [\underline{[gray]](x)} = T \text{ and } \underline{[[cat]](x)} = T$$

$$[\lambda x : x \in D_e . T iff [\lambda y : y \in D_e . T iff y \text{ is gray }](x) = T \text{ and } [\lambda y : y \in D_e . T iff y \text{ is a cat }](x) = T]$$

$$[NP_3] = [\lambda x : x \in D_e . \underline{[gray](x) = T} \text{ and } \underline{[cat](x) = T}] = [\lambda x : x \in D_e . [\lambda y : y \in D_e . y \text{ is gray }](x) = T \text{ and } [\lambda y : y \in D_e . y \text{ is a cat }](x) = T]$$

Since our functions, formulas, and proofs are only going to get longer, let's begin making moves towards abbreviated notations now.

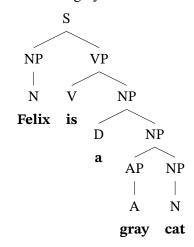
5 Deriving the Meaning of Sentences with Attributive Adjectives

We can now check to be sure that our semantic system correctly makes use our new compositional rule of Predicate Modification to the derive the correct truth-conditional statement.

Our syntactic assumptions regarding the sentence at hand are as follows:

(44) a. Felix is a gray cat.

b.



The lexical entries for the components parts of the sentence that are stored in the lexicon include:

(45) Lexical entries

a.
$$[Felix] = Felix$$

b.
$$[\![is]\!] = [\lambda f : f \in D_{\langle e,t \rangle} . f]$$

c.
$$[a] = [\lambda f : f \in D_{\langle e,t \rangle} . f]$$

d.
$$[gray] = [\lambda x : x \in D_e . x \text{ is gray }]$$

e.
$$[\![\operatorname{cat}]\!] = [\lambda x : x \in D_e . x \text{ is a cat }]$$

Our system currently employs the set of semantic rules listed below:

(46) Functional Application (FA)

If X is a node that has two daughters, Y and Z, and if $[\![Y]\!]$ is a function whose domain contains $[\![Z]\!]$, then $[\![X]\!] = [\![Y]\!] ([\![Z]\!])$.

(47) **Predicate Modification (PM)**

If X is a node that has two daughters, Y and Z, and if $[\![Y]\!]$ and $[\![Z]\!]$ are in $D_{\langle e,t\rangle}$, then $[\![X]\!] = [\lambda x : x \in D_e$. $[\![Y]\!](x) = T$ and $[\![Z]\!](x) = T$

(48) Non-Branching Nodes (NN) Rule

If X is a non-branching node that has Y as its daughter, then [X] = [Y]

(49) Terminal Nodes (TN) Rule

If X is a terminal node, then [X] is specified in the lexicon.

We can now see how the proof of the truth conditions can proceed with subproofs of the major constituents of the sentence.

Calculation of the Truth Conditions of Felix is a gray cat (50)

gray

- "Felix is a gray cat" is T iff (by syntax) ii. S is *T* iff (by notation) NP_1 VP N_1 NP_2 V Felix is NP_3 D a AP NP_4 Α N_4 cat
- $\llbracket S \rrbracket$ =Tiii.
- Calculation of $[\![NP_1]\!]$ iv.
 - a. $[NP_1]$ (by NN, TN)
 - b. Felix
- Calculation of [AP] v.
 - a. **[** AP **]** (by NN, TN)
 - b. $[\lambda x : x \in D_e . x \text{ is gray }]$
- Calculation of $[\![NP_4]\!]$ vi.
 - a. $[NP_4]$ (by NN, TN)
 - b. $[\lambda x : x \in D_e . x \text{ is a cat }]$
- vii. *Calculation of* $[\![NP_3]\!]$
 - a. $[NP_3]$ (by PM)
 - b. $[\lambda x : x \in D_e . [AP](x) = T \text{ and } [NP_4](x) = T]$ (by v.)
 - c. $[\lambda x : x \in D_e . [\lambda y : y \in D_e . y \text{ is gray }](x) = T \text{ and } [NP_4](x) = T] =$ (by LC, def.)
 - d. $[\lambda x : x \in D_e . x \text{ is gray and } [NP_4](x) = T]$ (by vi.)
 - e. $[\lambda x : x \in D_e . x \text{ is gray and } [\lambda y : y \in D_e . y \text{ is a cat }](x) = T]$ (by LC, def.) =
 - f. $[\lambda x : x \in D_e . x \text{ is gray and } x \text{ is a cat }]$

```
viii. Calculation of NP<sub>2</sub>
         a. [NP_2]
                                                                                         (by FA, LE, vii.)
         b. [a]([NP_3])
                                                                                         (by TN)
        c. [\lambda f : f \in D_{\langle e,t \rangle} . f]([\![ NP_3 ]\!])
                                                                                          (by LC)
         d. [NP_3]
                                                                                          (by vii.)
         e. [\lambda x : x \in D_e . x \text{ is gray and } x \text{ is a cat }]
ix.
         Calculation of [ VP ]
         a. VP
                                                                                          (by FA)
         b. \llbracket \text{ is } \rrbracket (\llbracket \text{ NP}_2 \rrbracket)
                                                                                          (by TN)
         c. [\lambda f : f \in D_{\langle e, t \rangle}, f]([\![ NP_2 ]\!])
                                                                                          (by LC)
         d. [NP_2]
                                                                                          (by viii.)
         e. [\lambda x : x \in D_e . x \text{ is gray and } x \text{ is a cat }]
X.
         \llbracket S \rrbracket
                                                                            = T iff (by FA, ix., iv.)
         [VP]([NP_1])
                                                                            = T iff (by iv.)
xi.
                                                                            = T iff (by ix.)
xii.
        ¶ VP ∥(Felix)
xiii. [\lambda x : x \in D_e . x \text{ is gray and } x \text{ is a cat }](\text{Felix}) = T \text{ iff} \text{ (by LC)}
xiv.
        Felix is gray and Felix is a cat
                                                                                          (nailed it)
```

Thus, (50i) iff (50xiv), or "Felix is a gray cat" is T iff Felix is gray and Felix is a cat.

By repeating the process made explicit in this handout, we would find that our rule of Predicate Modification will work for a range of **intersective** adjectives that appear in attributive position and derive seemingly correct truth-conditional statements:

- (51) a. "Felix is a **furry dog**" is *T iff* Felix is **furry** and Felix is a **dog**.
 - b. "Felix is a **single man**" is *T iff* Felix is **single** and Felis is a **man**.
 - c. "Felix is a **bald hamster**" is *T iff* Felix is **bald** and Felix is a **hamster**.

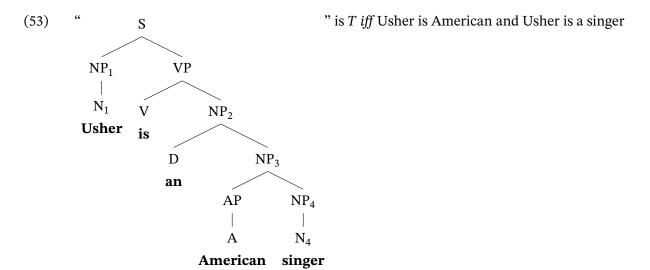
There are, however, numerous adjectives for which this doesn't deliver accurate truth conditions:

- (52) a. ?? "Richard is a **tall jockey**" is *T iff* Richard is **tall** and Richard is a **jockey**.
 - b. ?? "Barack is a **young ex-president**" is *T iff* Barack is **young** and Barack is a **an ex-president**.
 - c. ?? "Proxima Centauri is a **cold star**" is *T iff* P.C. is **cold** and P.C. is a **star**.

We'll consider the semantics of these **subsective** adjectives during our next meeting.

6 Practice

Exercise. Please provide a semantic proof of the truth-conditional statement below:



In order to do this, we will need to provide the extensions of the lexical items and provide a step-by-step proof of these reported truth conditions.